Oblivious Monitoring for Discrete-Time STL via Fully Homomorphic Encryption

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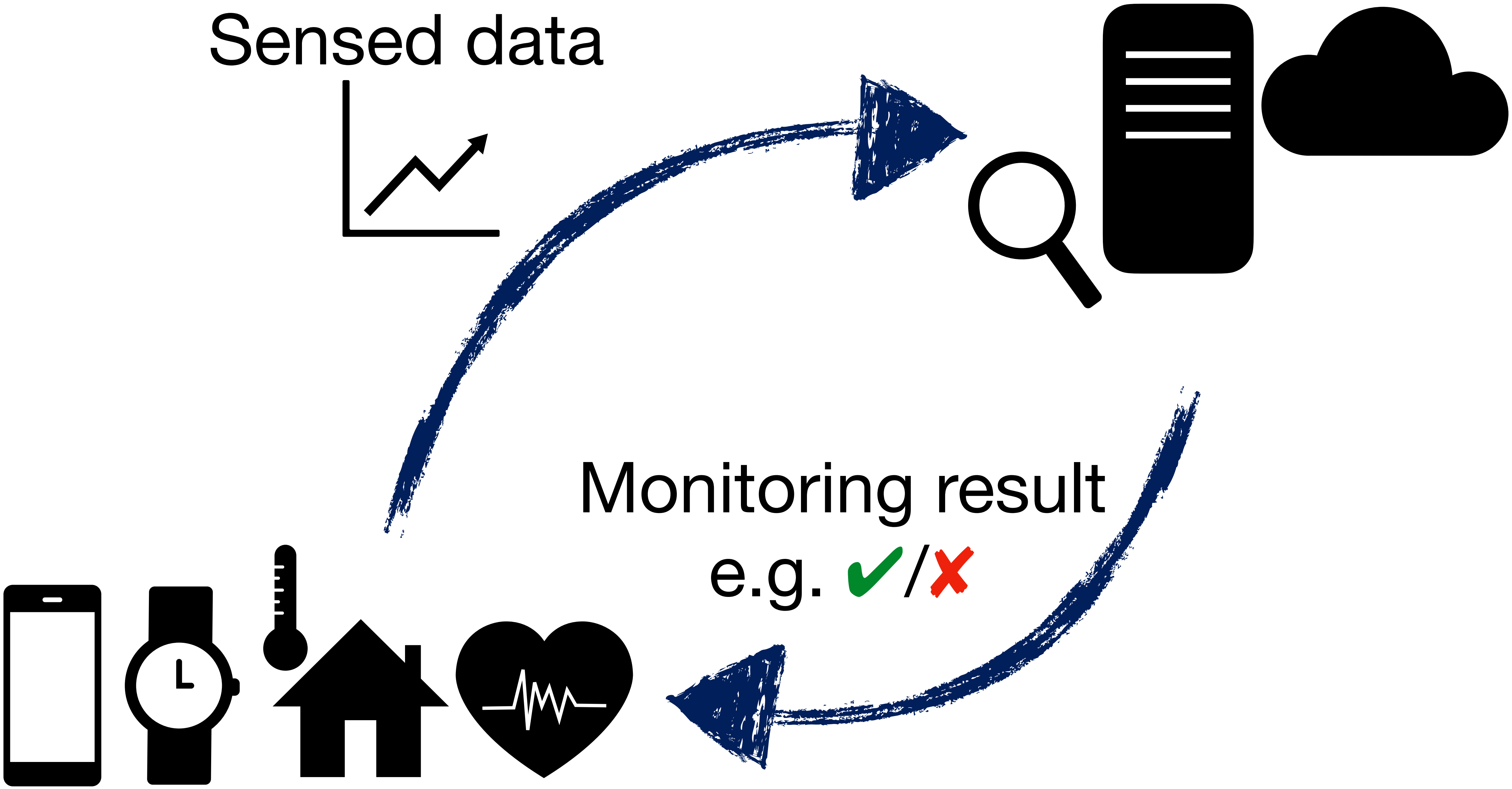
Kyoto University[1], Cybozu, Inc.[2], Beihang University[3]

RV 2024, 15th Oct. 2024

Monitoring with IoT is useful



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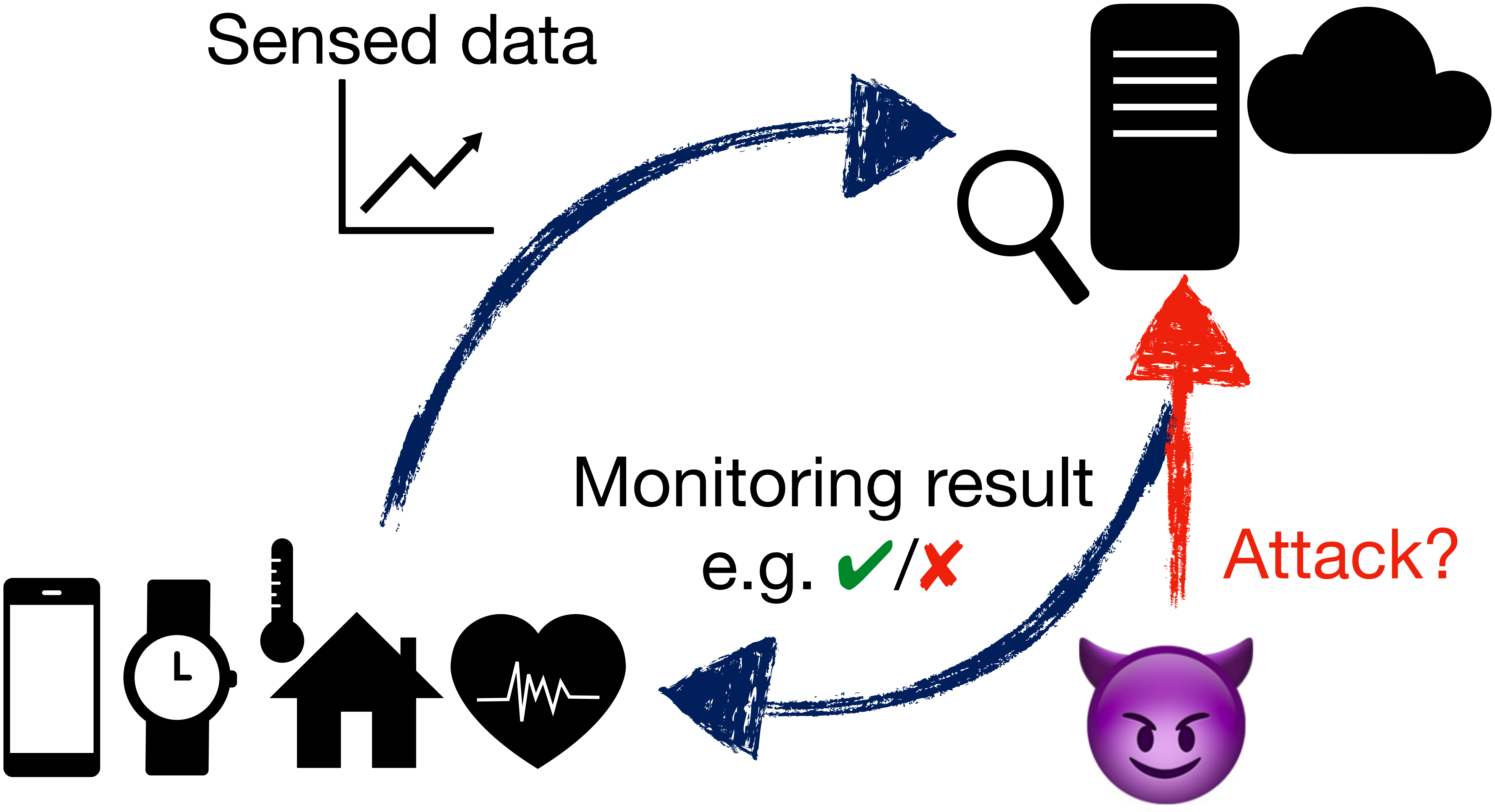

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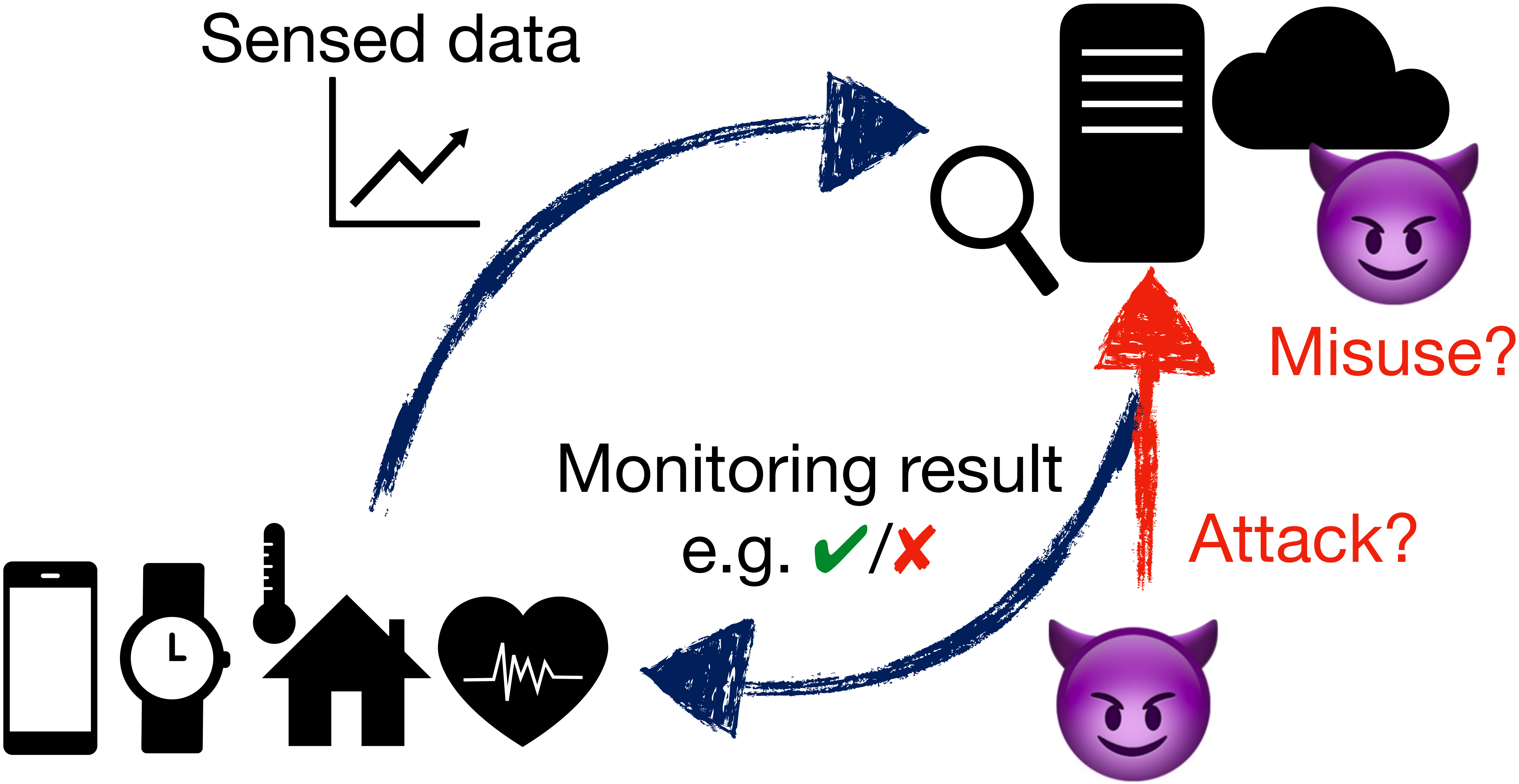
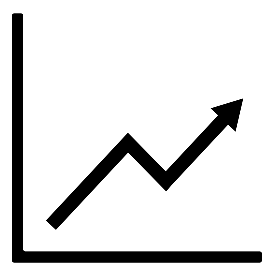
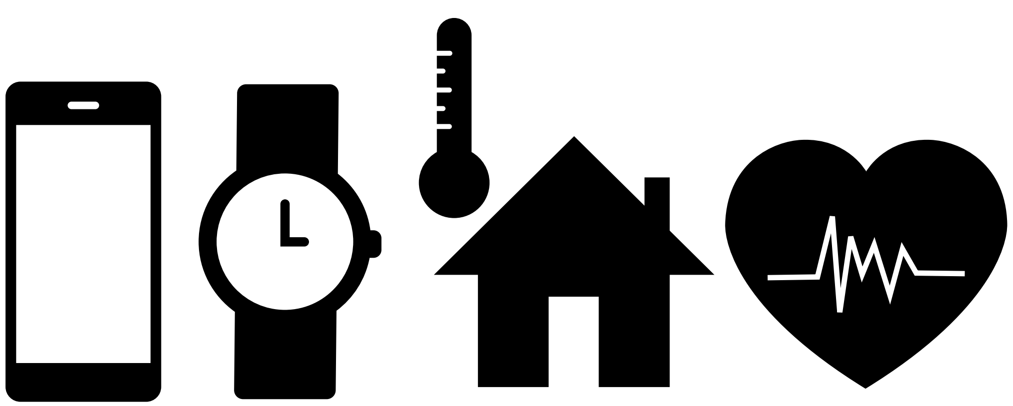


Monitoring with IoT is useful
...but privacy?

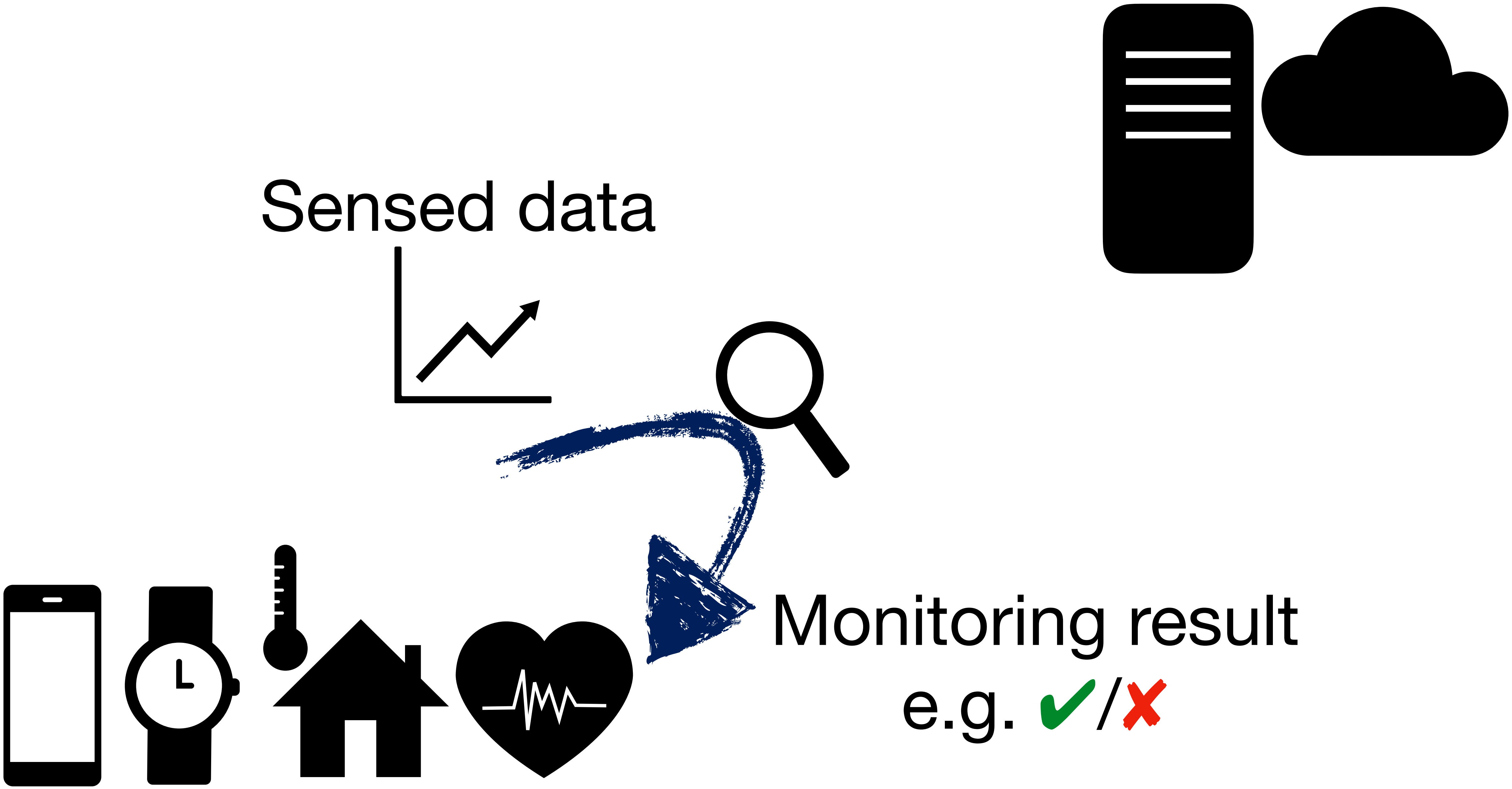
Sensed data





Monitoring result
e.g. ✔/✘

Monitoring with IoT is useful
...but privacy?



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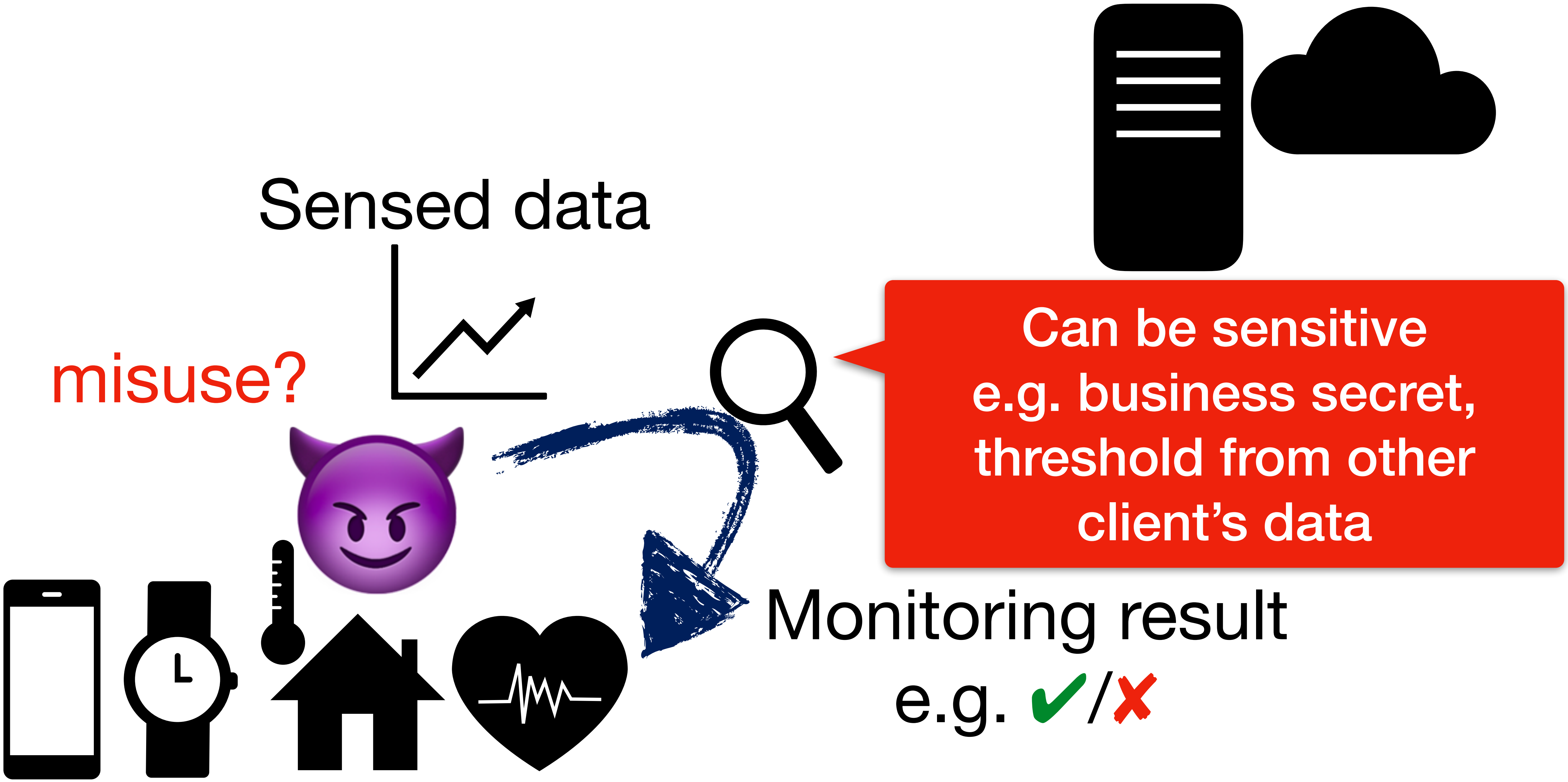

Q. Can server monitor without knowing the content?

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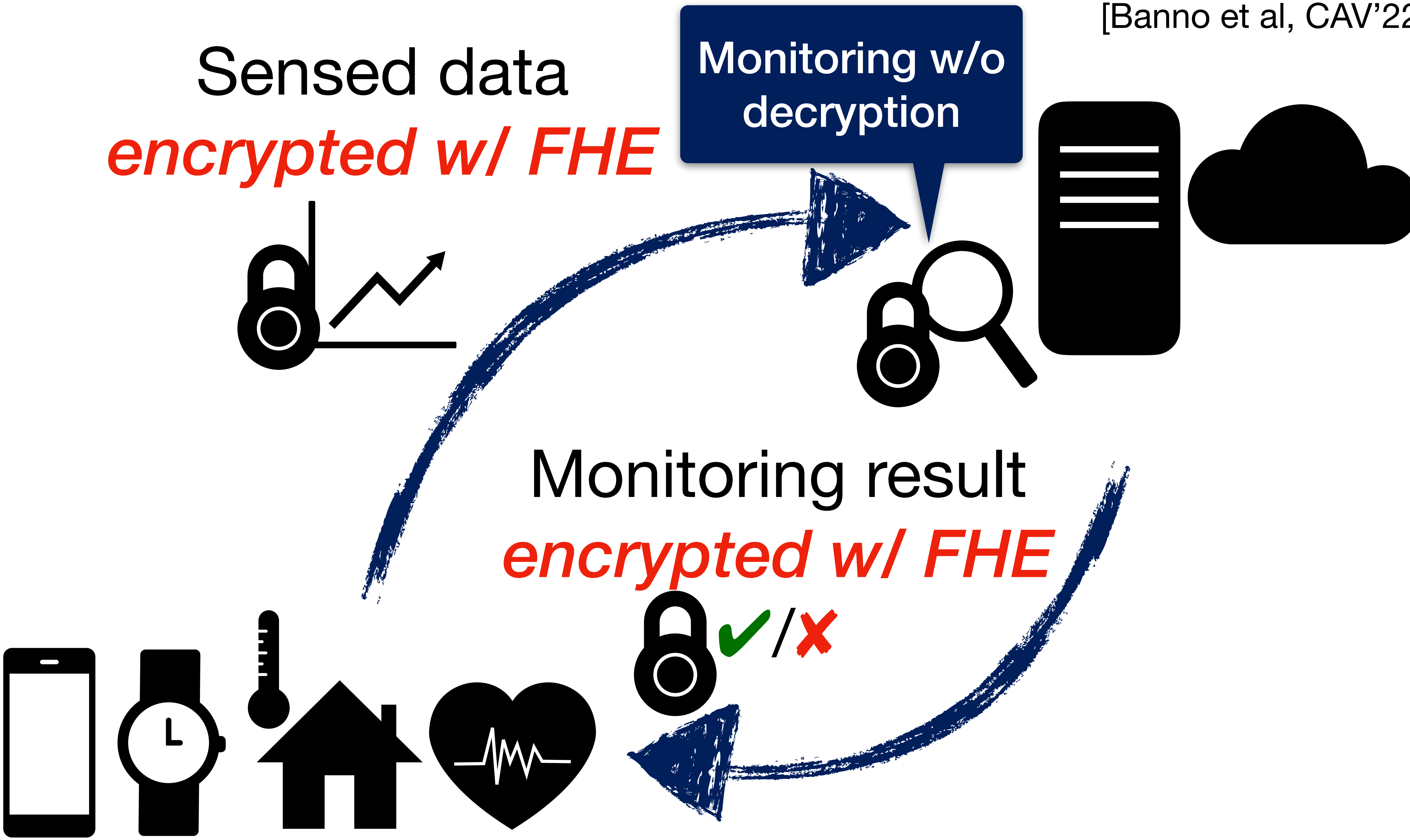
A. Yes, with Fully Homomorphic Encryption

Oblivious Online LTL Monitoring

[Banno et al, CAV'22]



Oblivious Online LTL Monitoring

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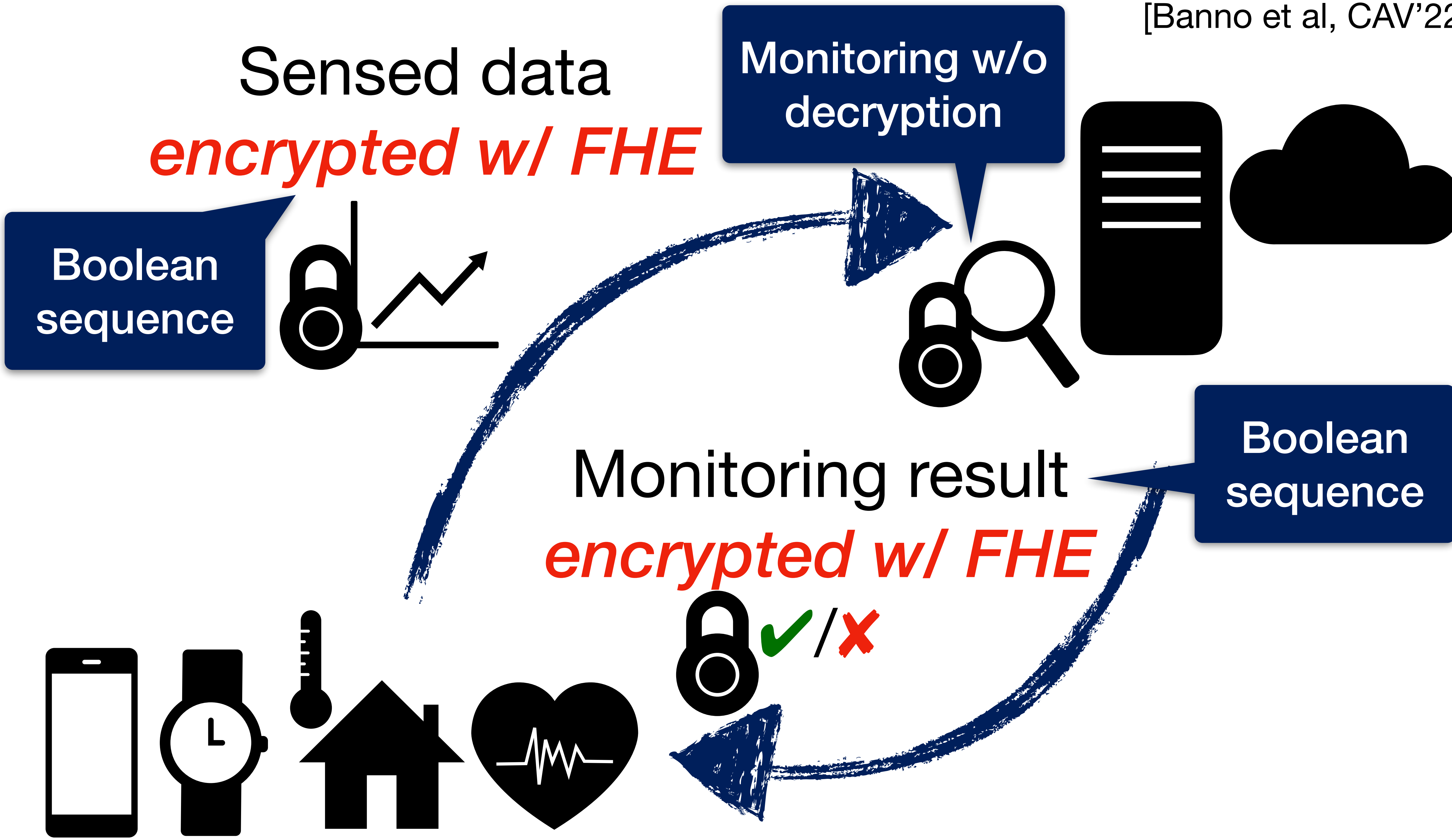

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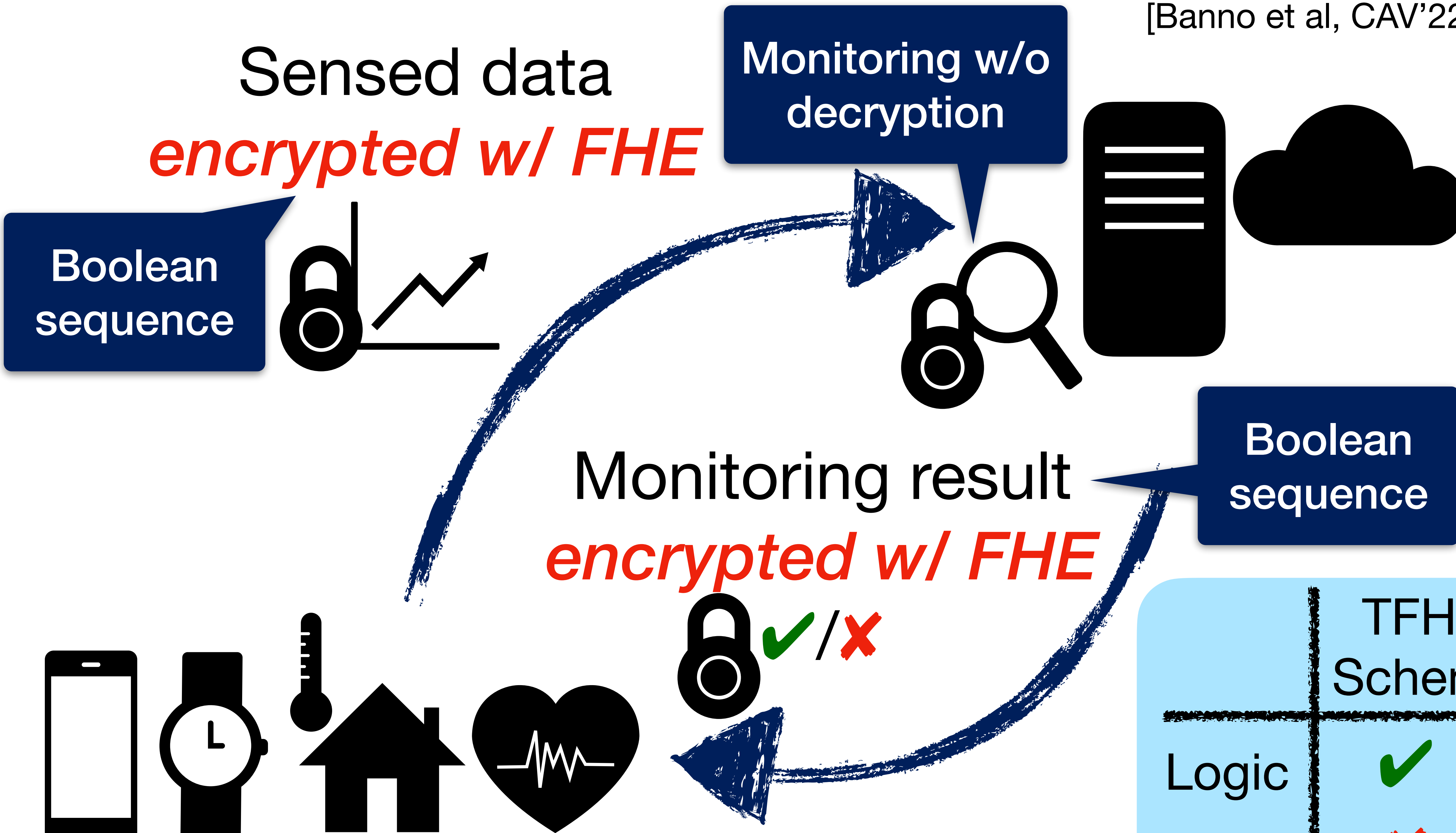

	TFHE Scheme
Logic	✔
Arith.	✘

Oblivious Online LTL Monitoring

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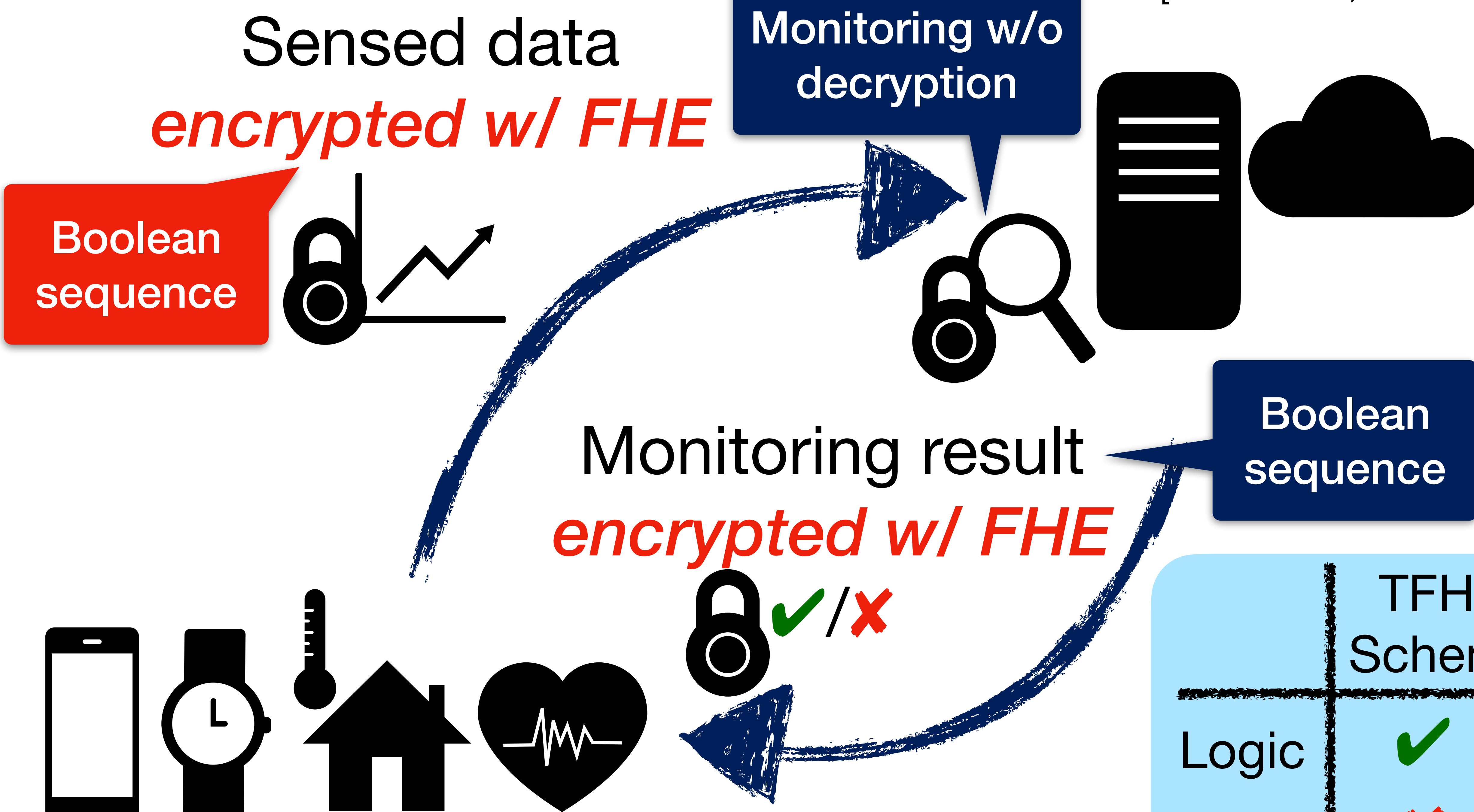

	TFHE Scheme
Logic	✔
Arith.	✘

Q. Can we handle temporal + arith?

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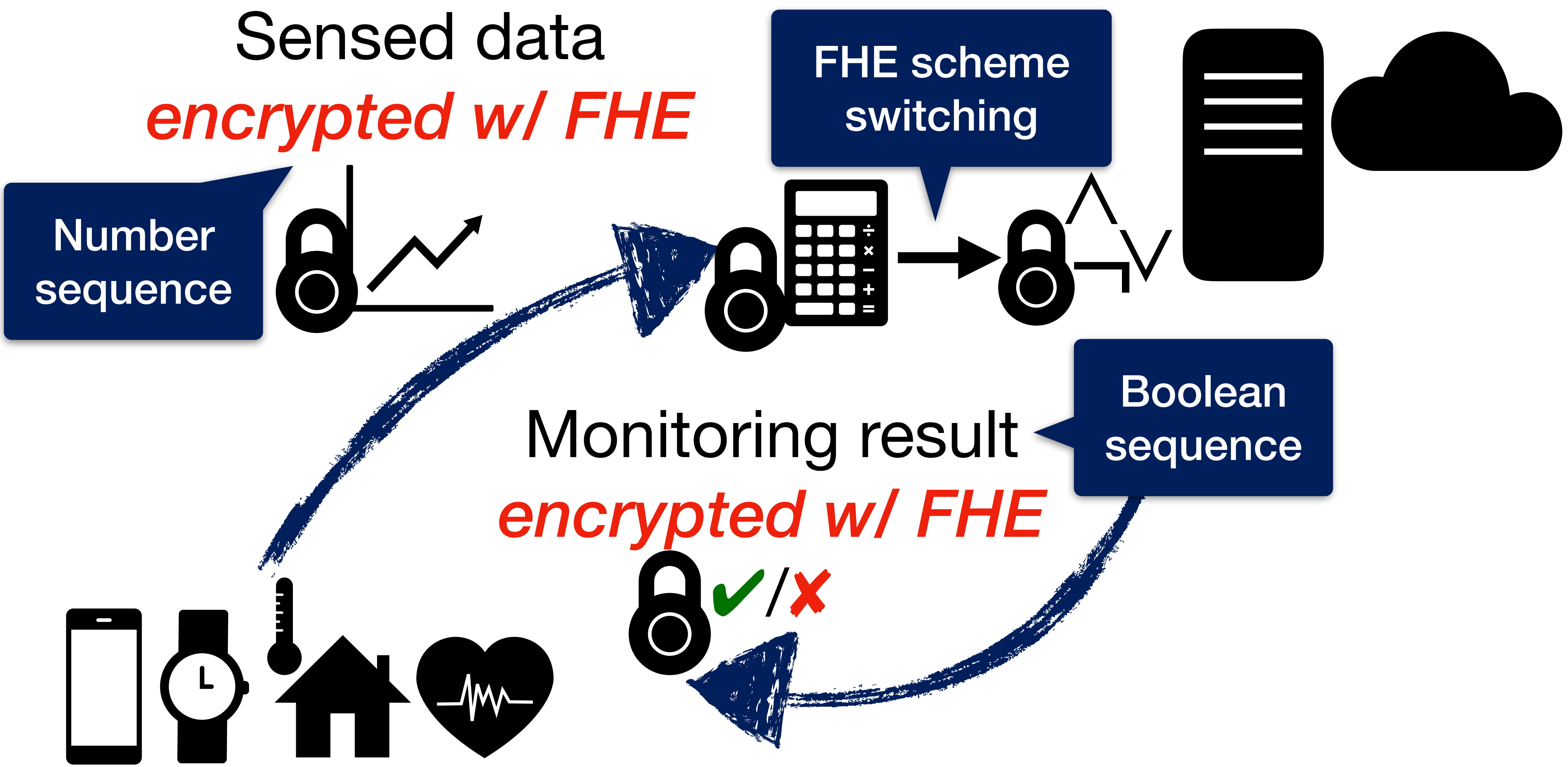
A. Yes, by bridging FHE schemes

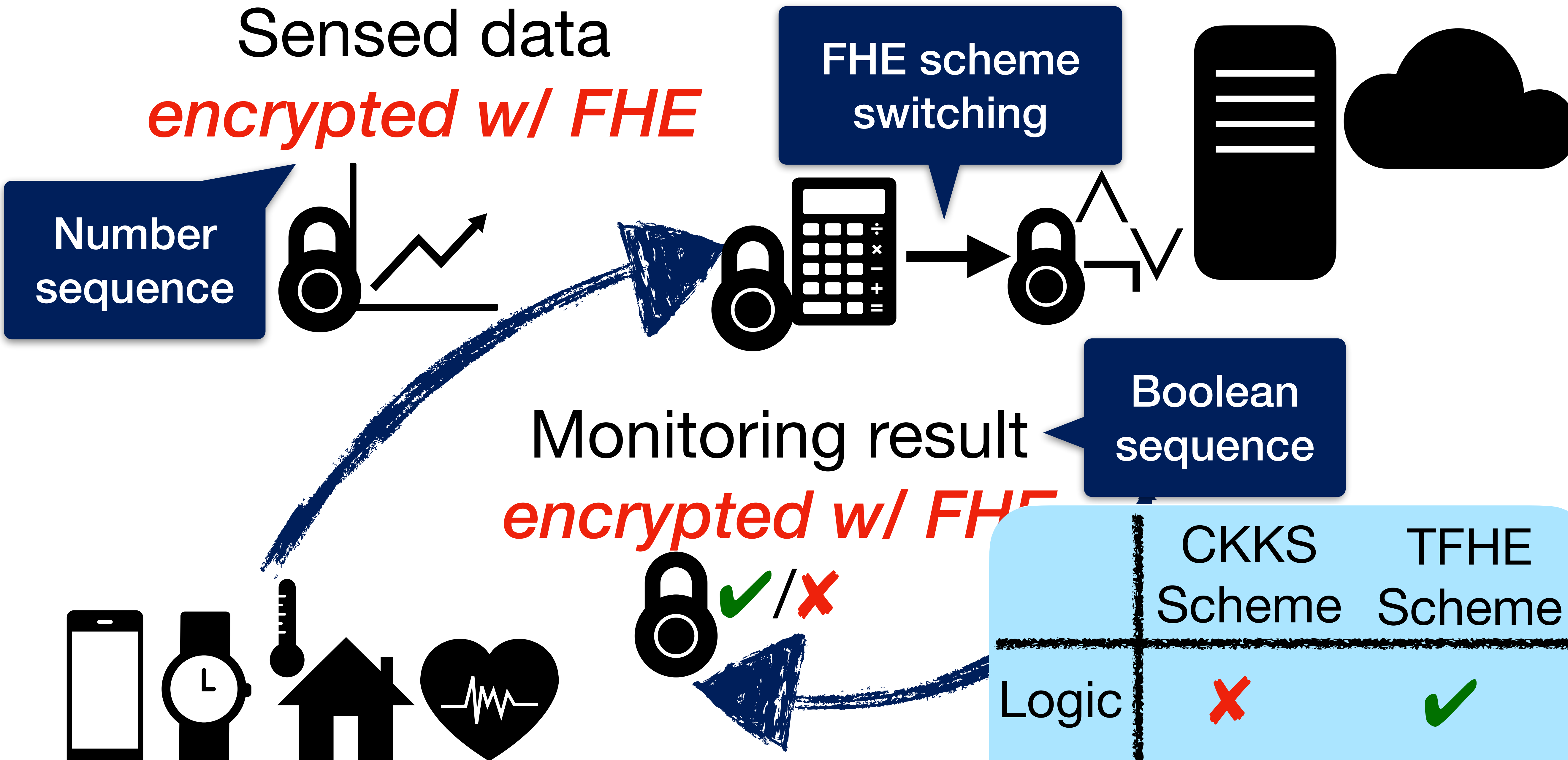
Oblivious Online STL Monitoring

[Contribution]



Oblivious Online STL Monitoring

[Contribution]



	CKKS Scheme	TFHE Scheme
Logic	✘	✔
Arith.	✔	✘
⋮		

Contributions

- Online oblivious discrete-time STL monitoring protocol
  - Combining CKKS and TFHE schemes
  - Note: discrete-time but with (linear) arith. predicates
- Optimization of scheme switching for STL monitoring
  - The largest technical contribution
- Implementation + experiments
  → Fast enough for blood glucose monitoring
  Works for RSS monitoring, too

Outline

- Review: Oblivious LTL monitoring with FHE
- Oblivious discrete-time STL monitoring with FHE
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Fully Homomorphic Encryption

Normal Encryption (e.g. RSA) | **Fully Homomorphic Encryption**

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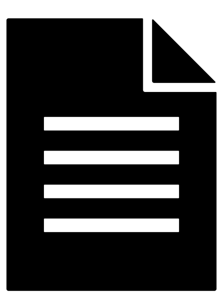

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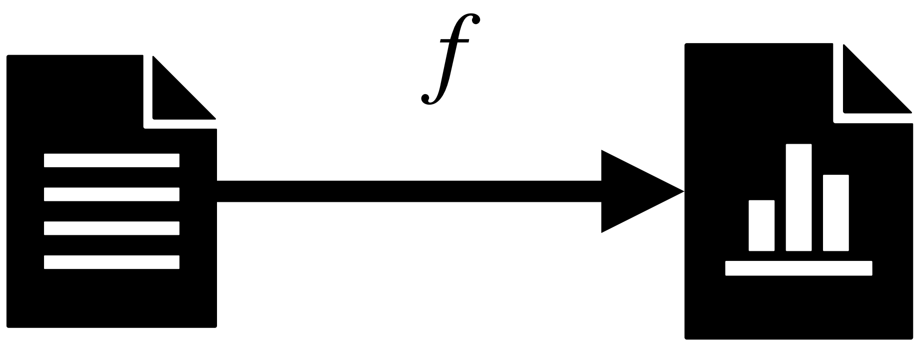

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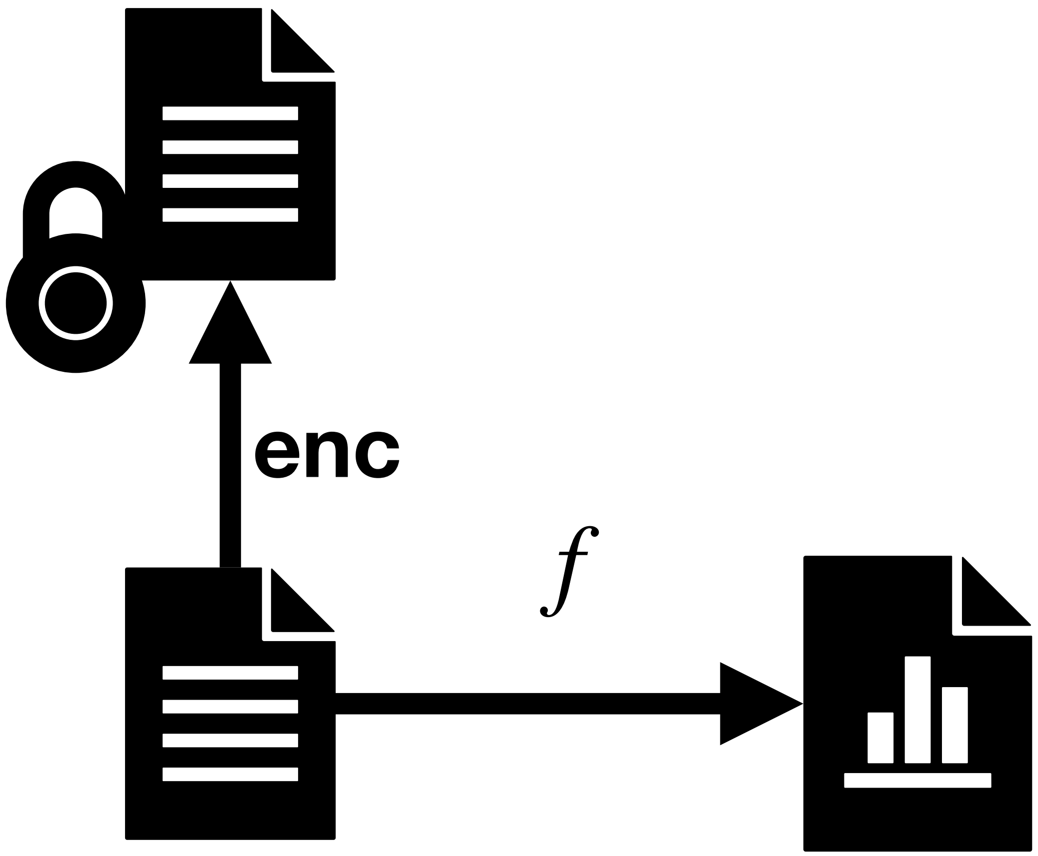

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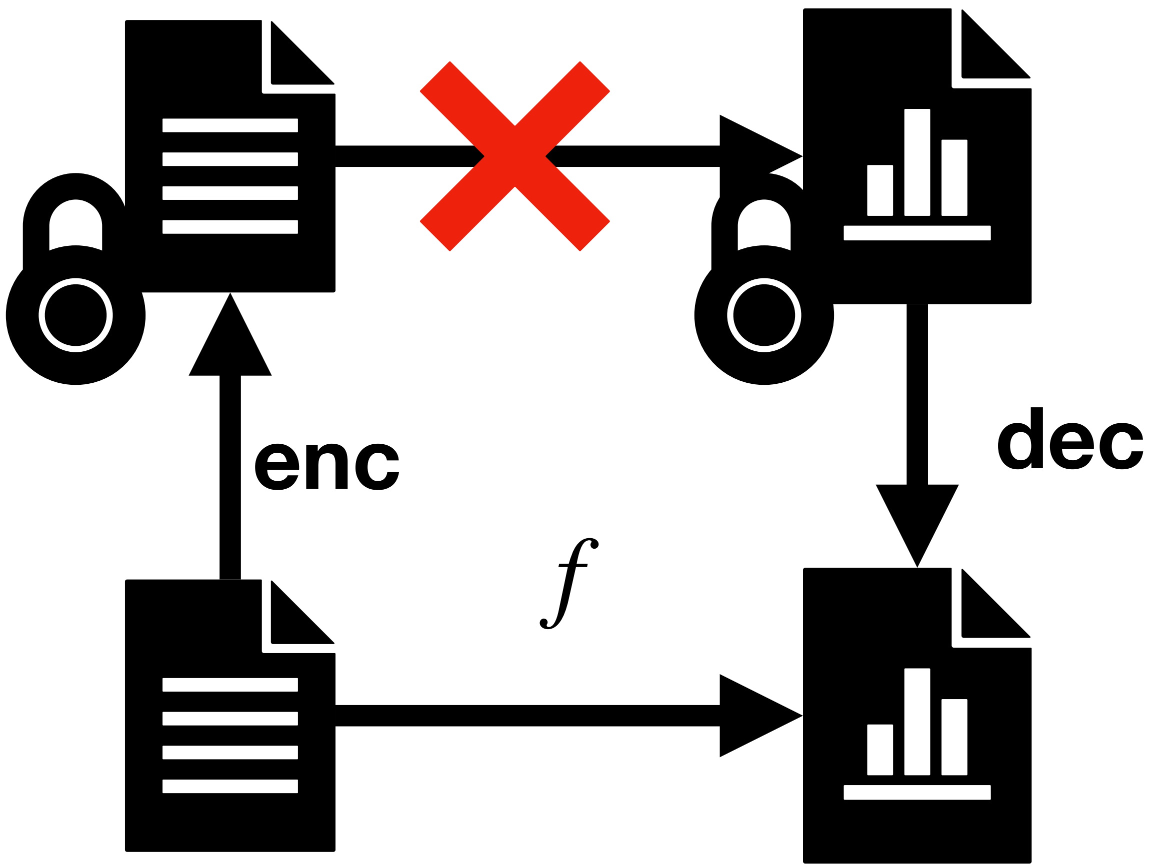

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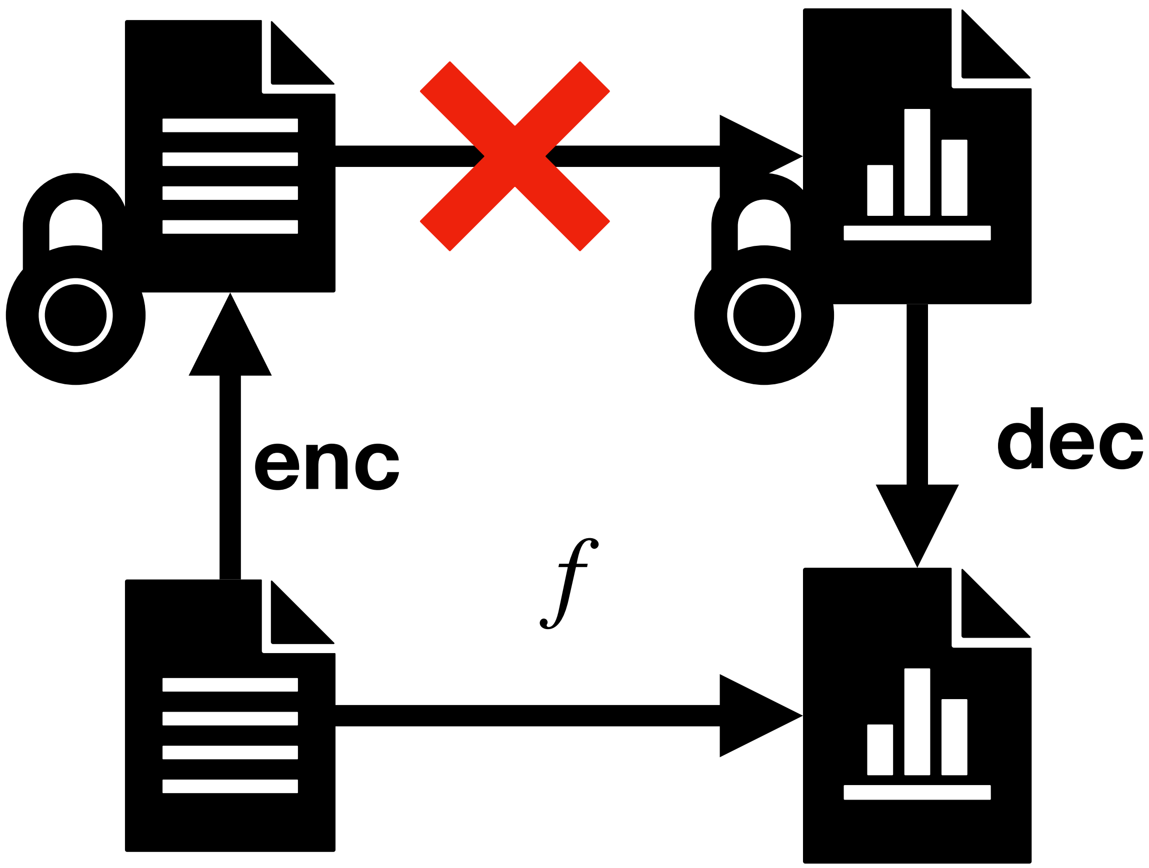

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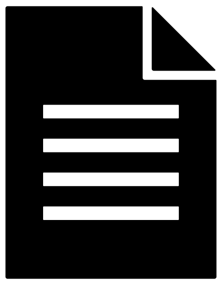

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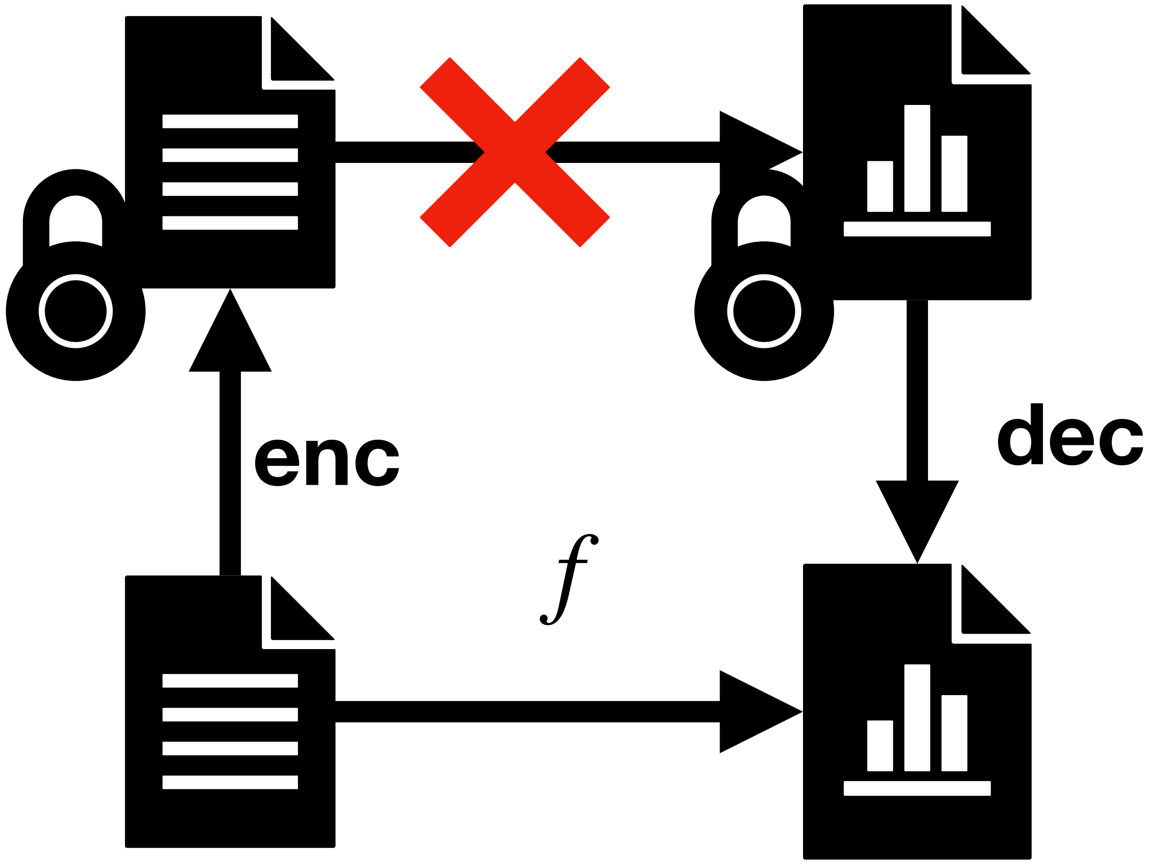

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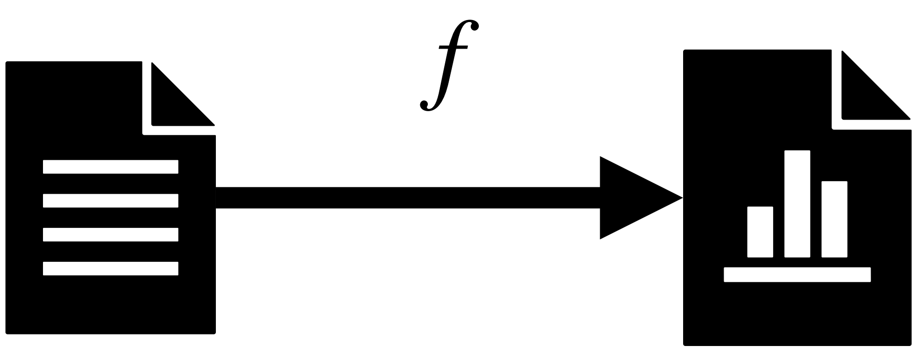

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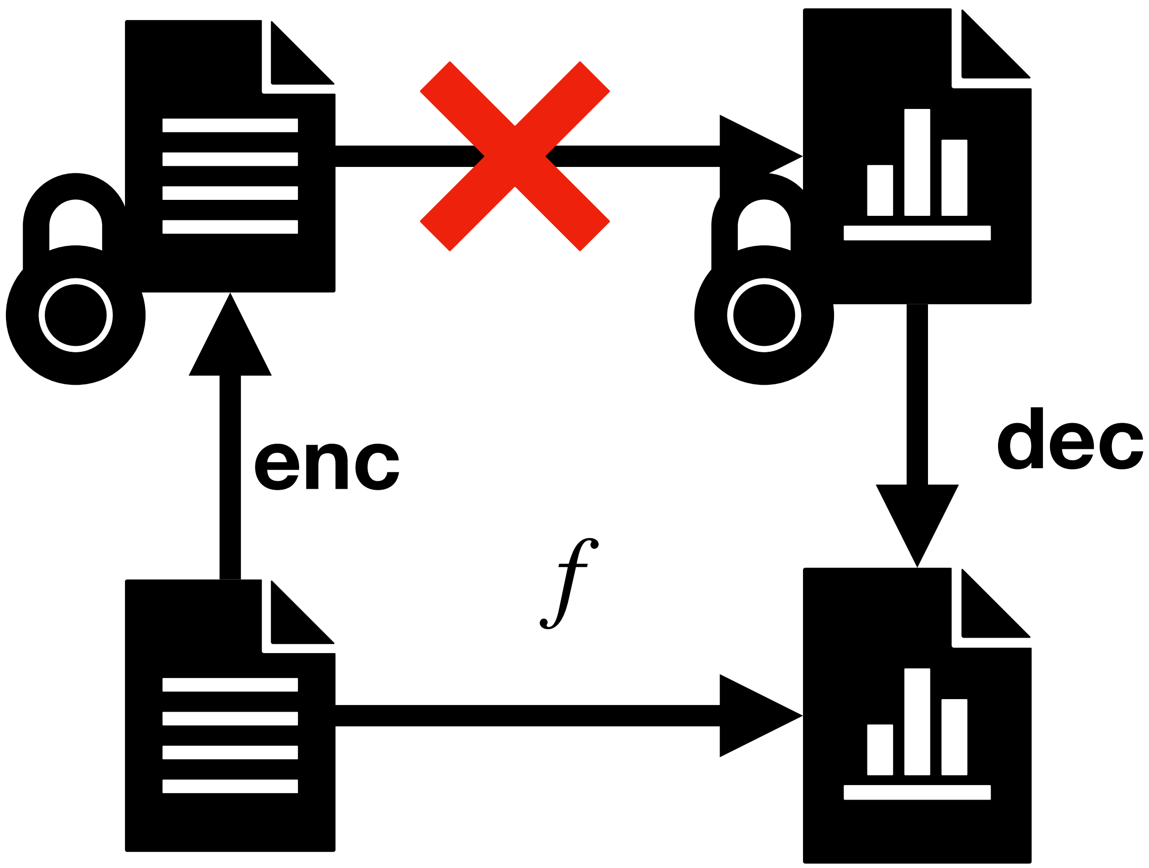

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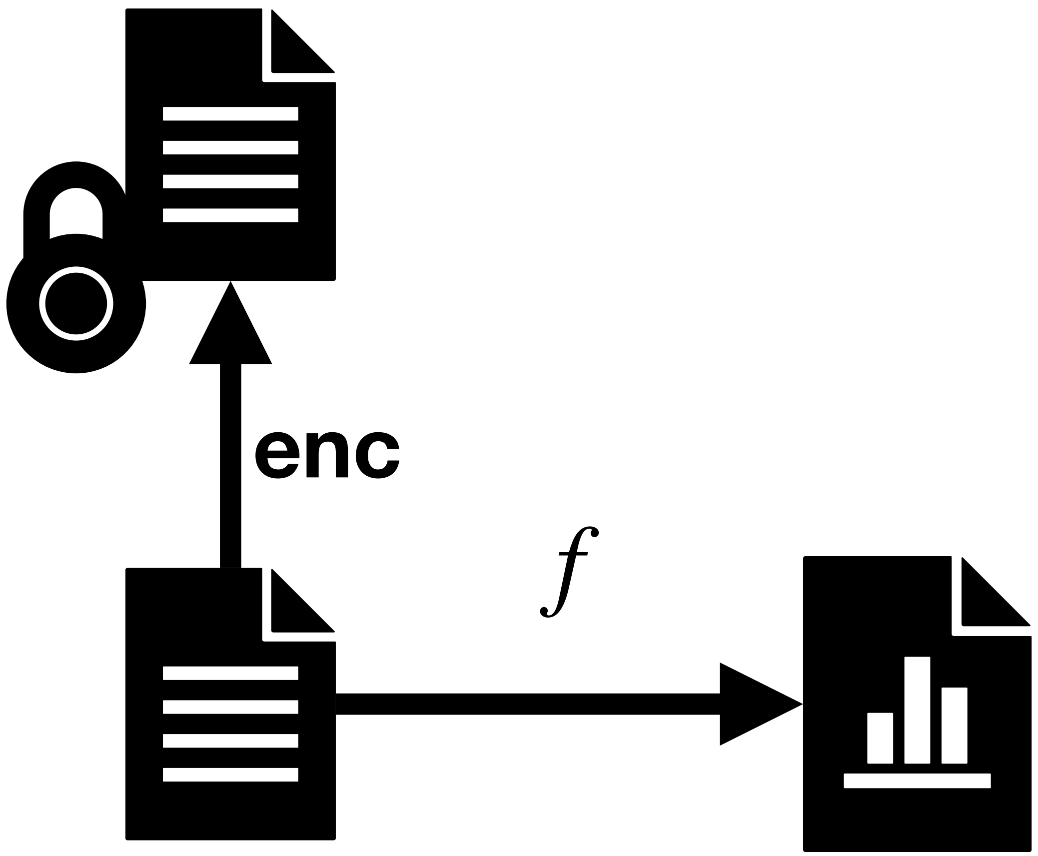

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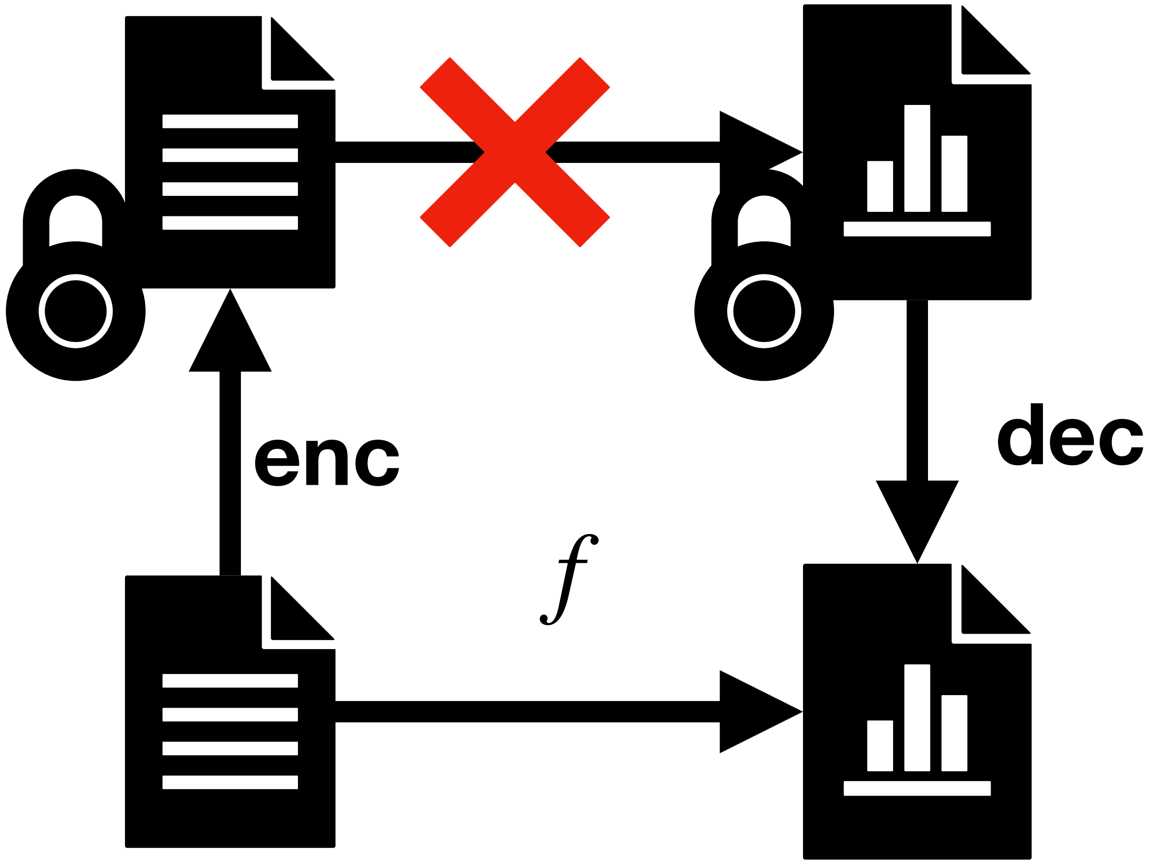

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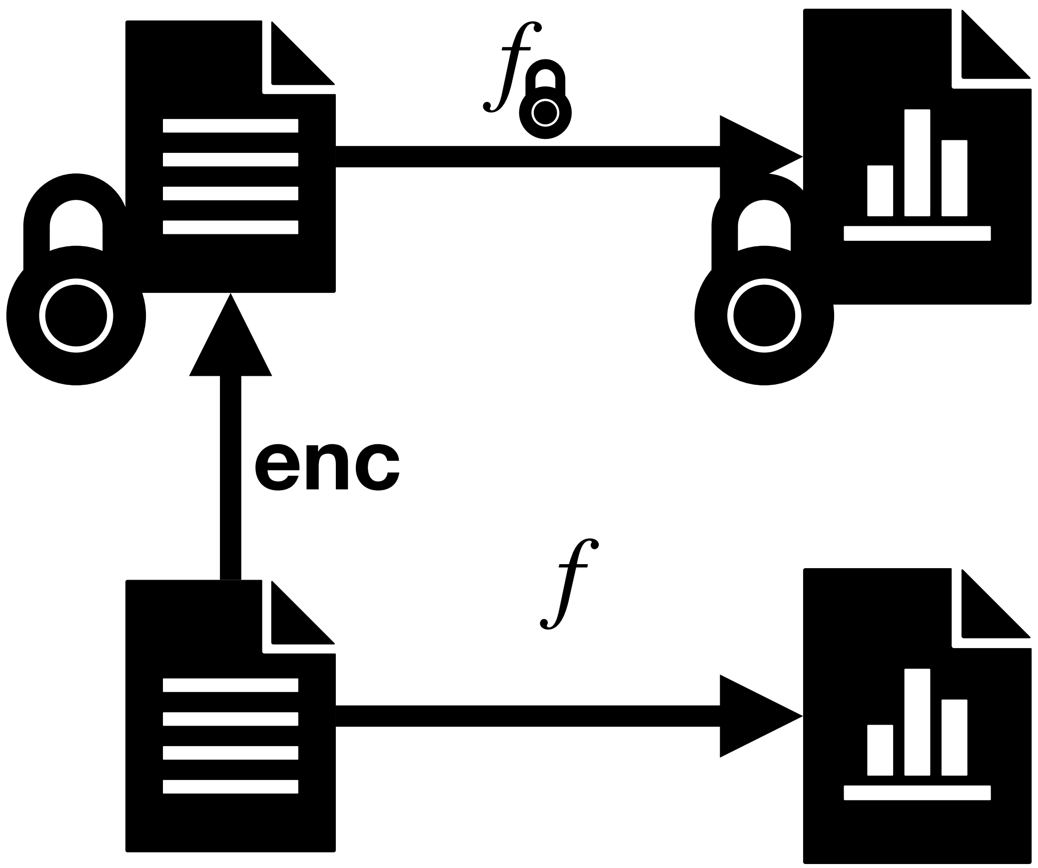

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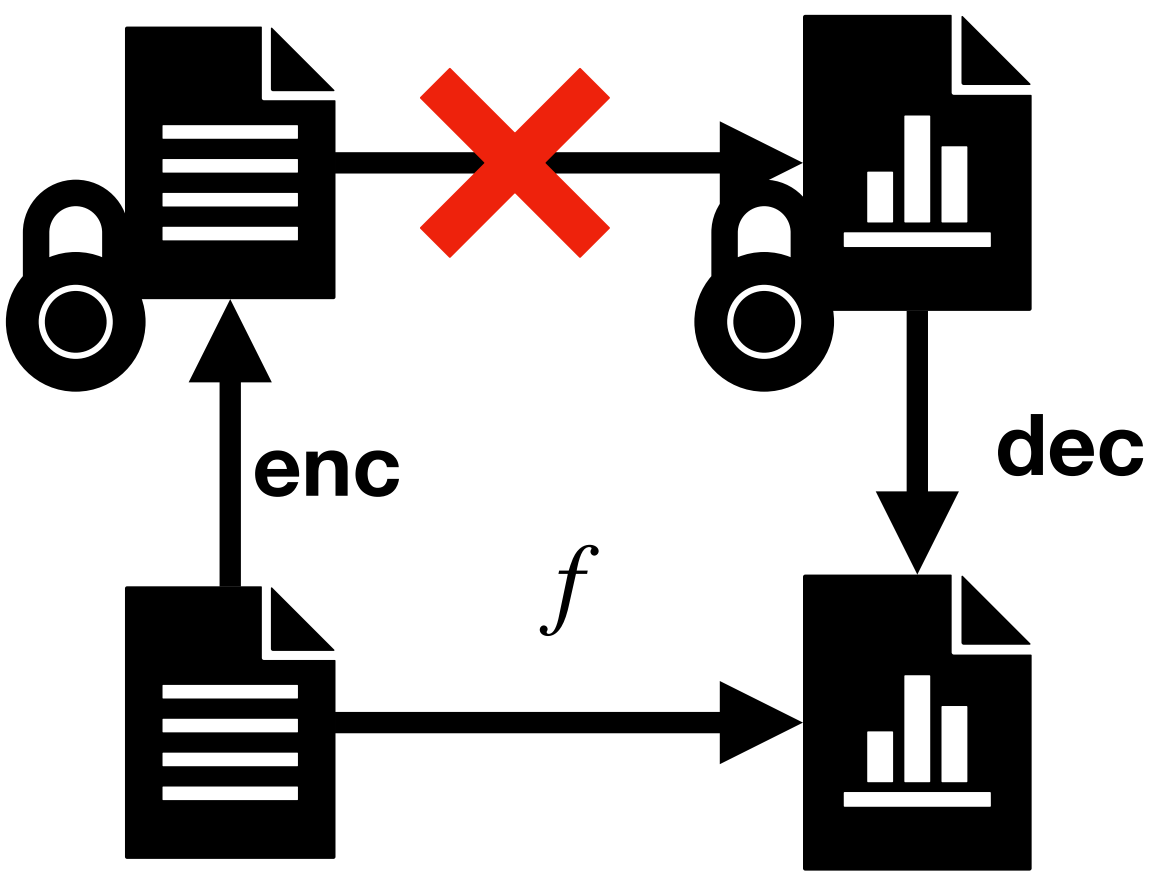

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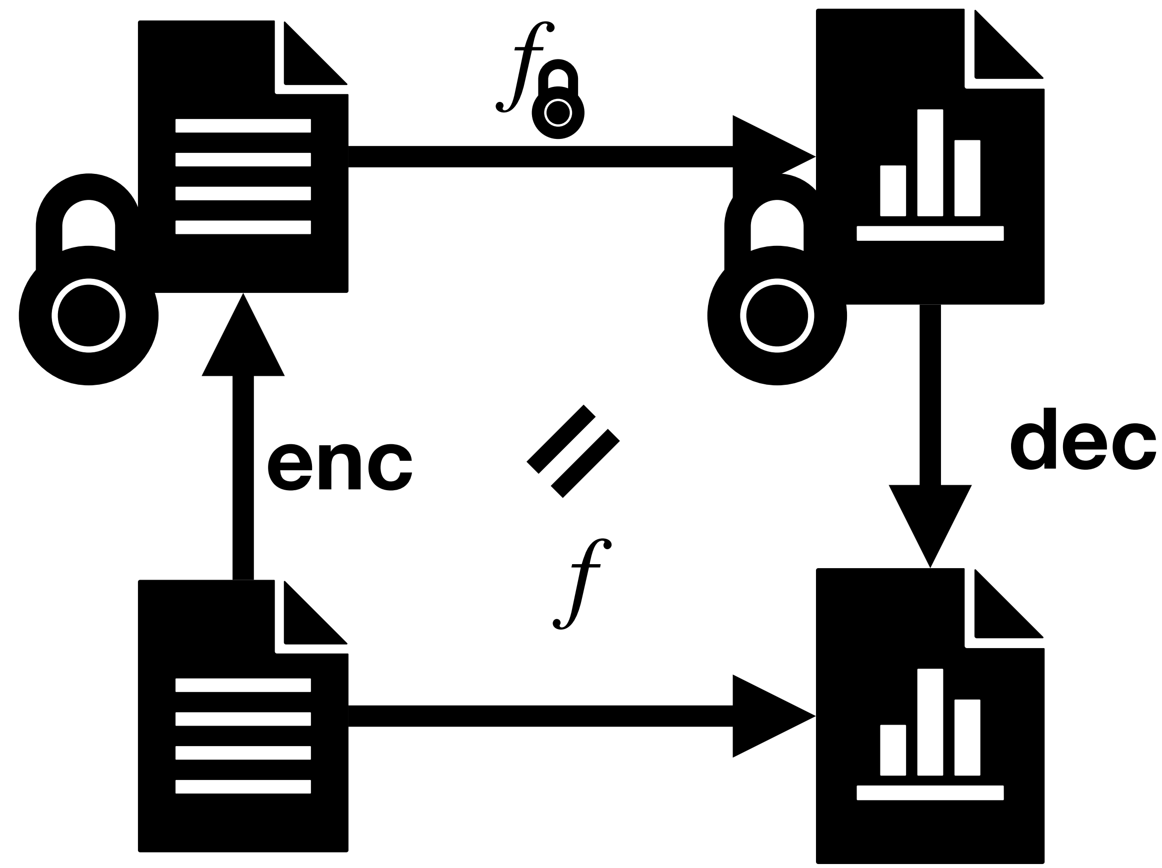

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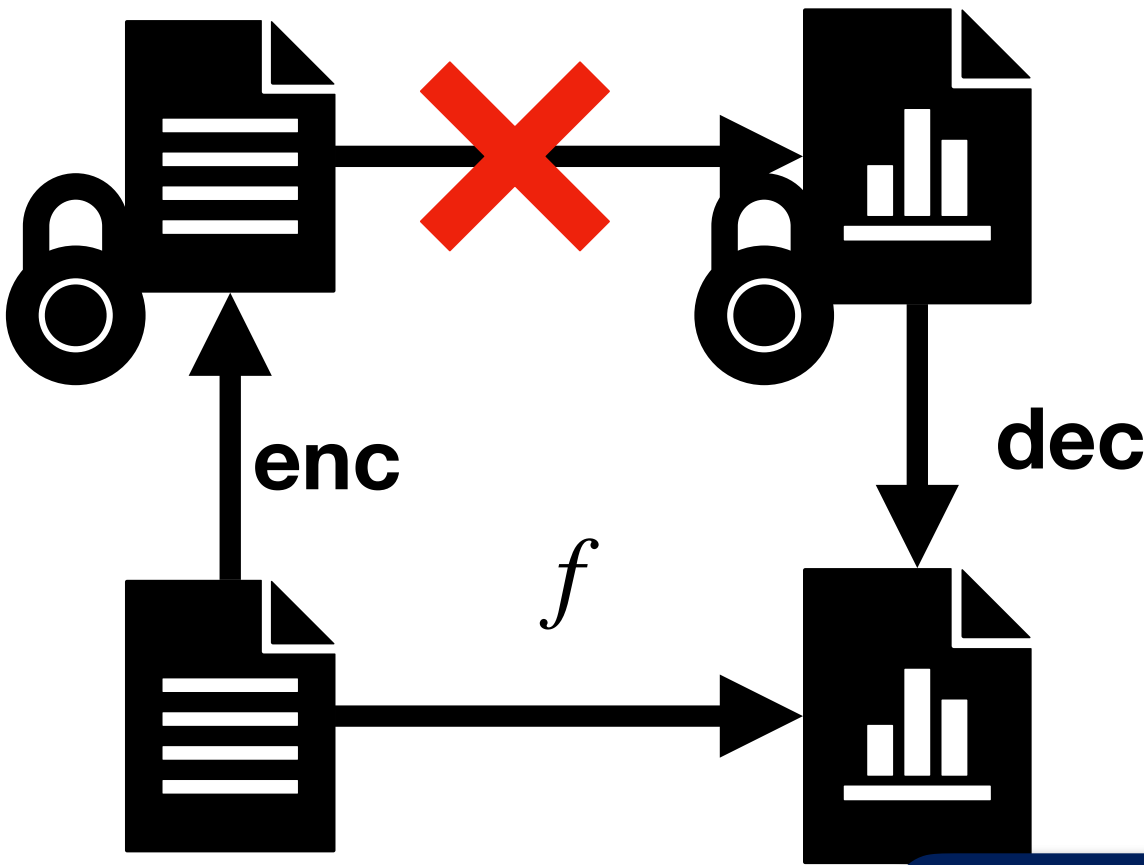

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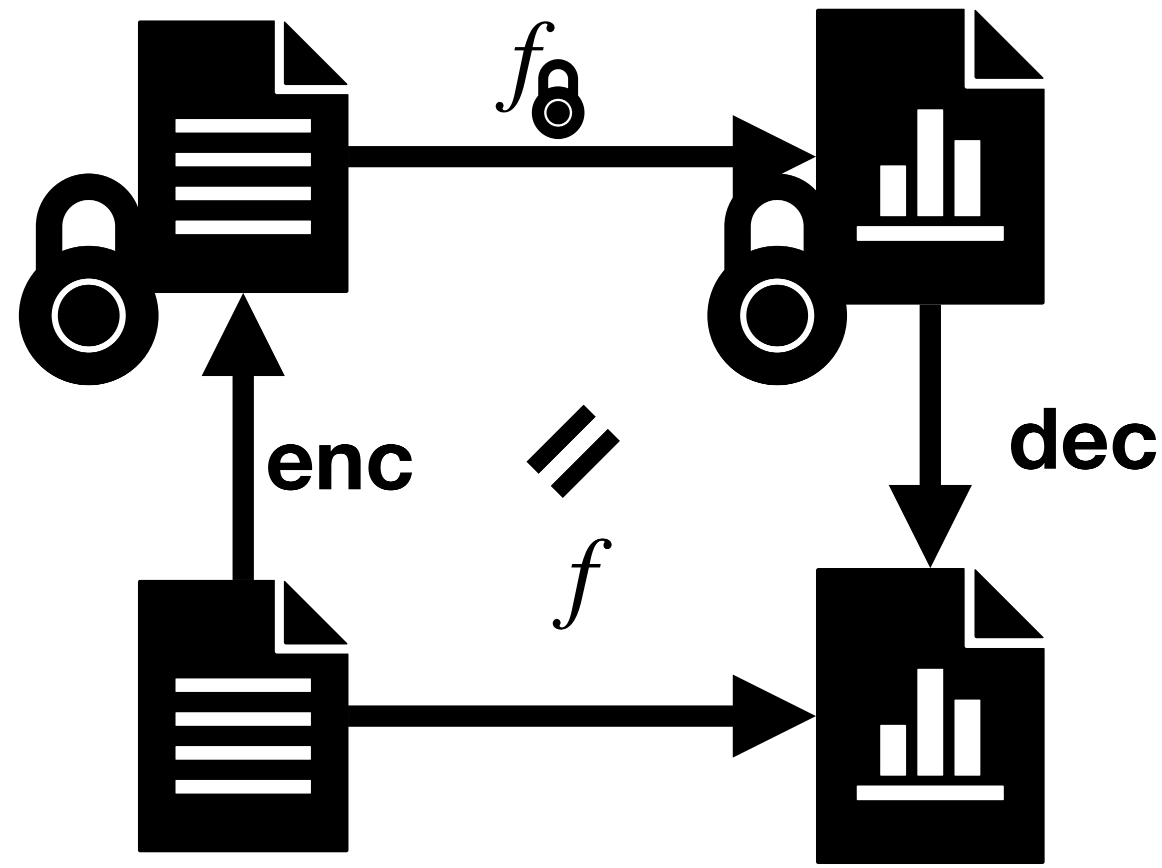

Remark: Naive computation is usually very slow
→ dedicated algorithm is necessary
e.g. VM with FHE is a few Hz

Ciphertexts in FHE: (R)LWE

Noisy ciphertexts/operations for security

- (Low-level) Ciphertext: (masked) message + “noise”
- (Noisy) Operations
  - Noisy encryption
  - Noisy addition
  - Noisy multiplication
  - …
- Special Operation: Bootstrapping

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Increases noise
→ Eventually breaks message

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Reduces noise

Ciphertexts in FHE: (R)LWE

Noisy ciphertexts/operations for security

- (Low-level) Ciphertext: (masked) message + “noise”

> Result is always approx.
> → We need high-level “scheme”

- (Noisy) Operations
  - Noisy encryption
  - Noisy addition
  - Noisy multiplication
  - …

> Increases noise
> → Eventually breaks message

- Special Operation: Bootstrapping

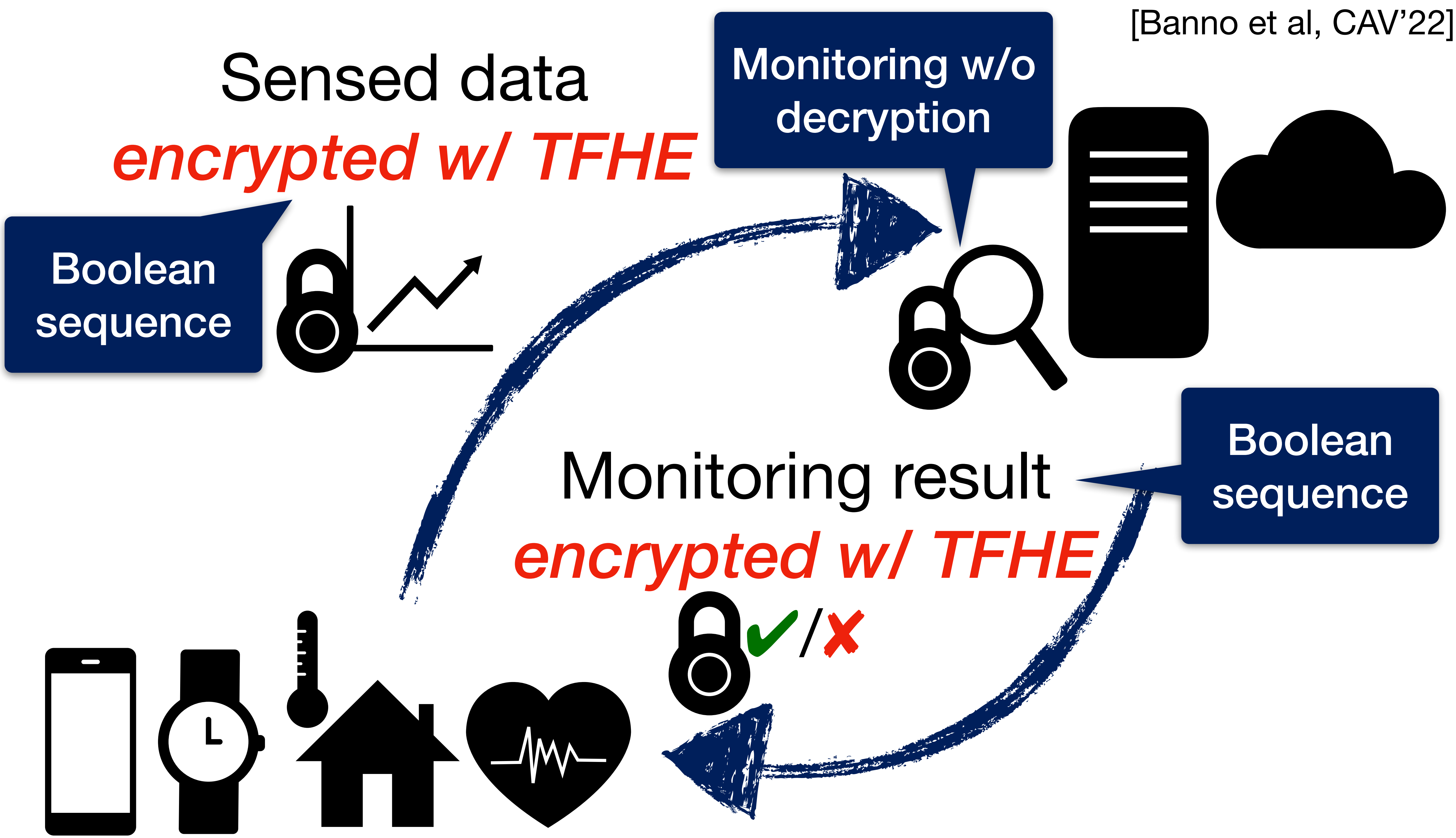
> Reduces noise

CKKS vs. TFHE Schemes

	CKKS [Cheon et al., ASIACRYPT'17]	TFHE [Chillotti et al., J. Crypto '20]
Typical Usage	Approx. Numbers w/ (linear) Arith. Op. e.g. +, -, *	(Exact) Booleans with logical operations e.g. and, or, not, nand
Bootstrapping for noise reduction	Very Slow e.g. > 20 sec. Can be ≈ 1.5 min.	(Relatively) Fast e.g. < 500 ms.

Values are taken from Al Badawi, Ahmad, and Yuriy Polyakov. "Demystifying bootstrapping in fully homomorphic encryption." Cryptology ePrint Archive (2023).

Oblivious Online LTL Monitoring



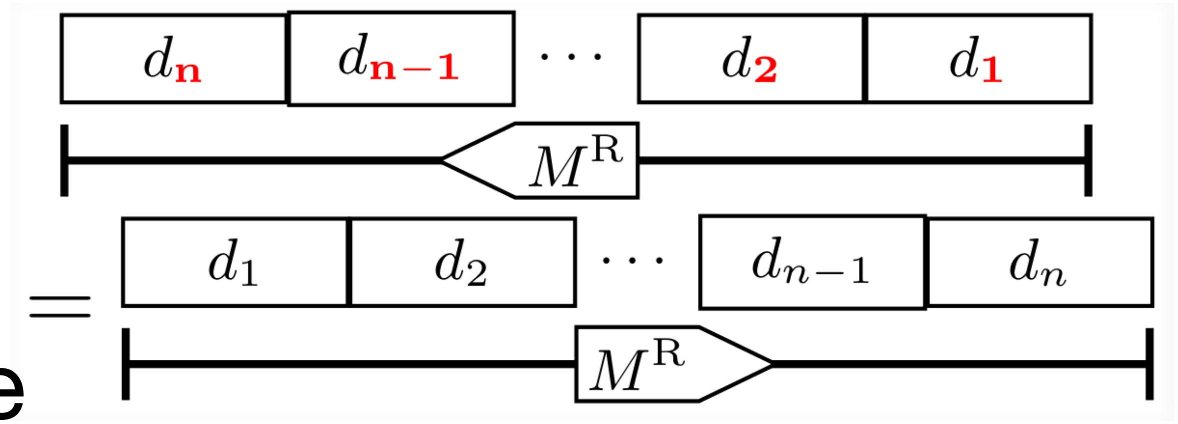
Oblivious Online DFA Execution

[Banno et al, CAV'22]

Two algorithms for FHE-based DFA execution

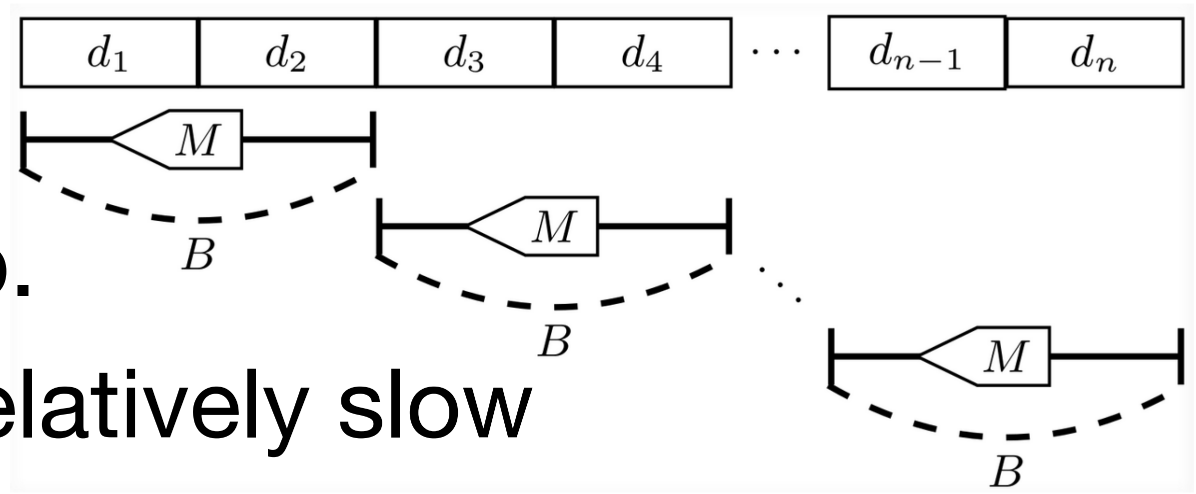
Reverse

- Reverse the given DFA
- Reversed DFA can be huge



Block

- "Blocked" backward comp.
- Jumping to next block is relatively slow



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Discrete-time STL: LTL + arith.

$$\varphi, \varphi' ::= \mu > c \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathscr{X}\varphi' \mid \varphi \mathscr{U}_{[i,j)} \varphi'$$

Discrete-time STL: LTL + arith.

$$\varphi, \varphi' ::= \mu > c \mid \neg\varphi \mid \varphi \lor \varphi \mid \mathscr{X}\varphi' \mid \varphi\,\mathscr{U}_{[i,j)}\varphi'$$

(Linear) arith. predicate

Same as LTL
→ Can be converted to a DFA

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# of arith. op. in μ is known (to the server)
→ Bootstrapping is unnec. for appropriate param.

Log length is unbounded
→ Need bootstrapping

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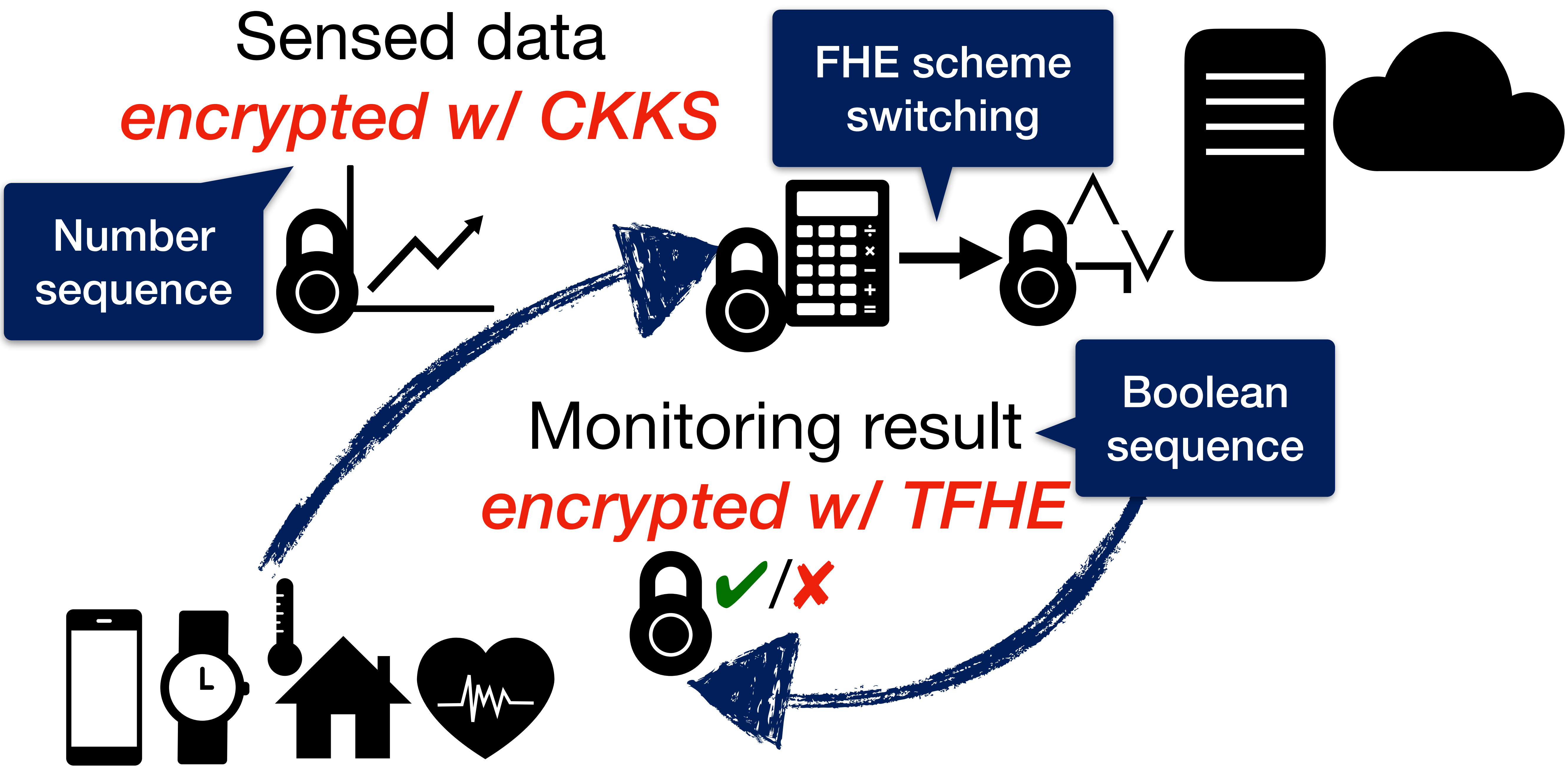
	CKKS [Cheon et al., ASIACRYPT'17]	TFHE [Chillotti et al., J. Crypto '20]
Typical Usage	Numbers	Booleans
Bootstrapping	Very Slow	(Relatively) Fast

Oblivious Online Monitoring of STL with arithmetic predicates [Contribution]

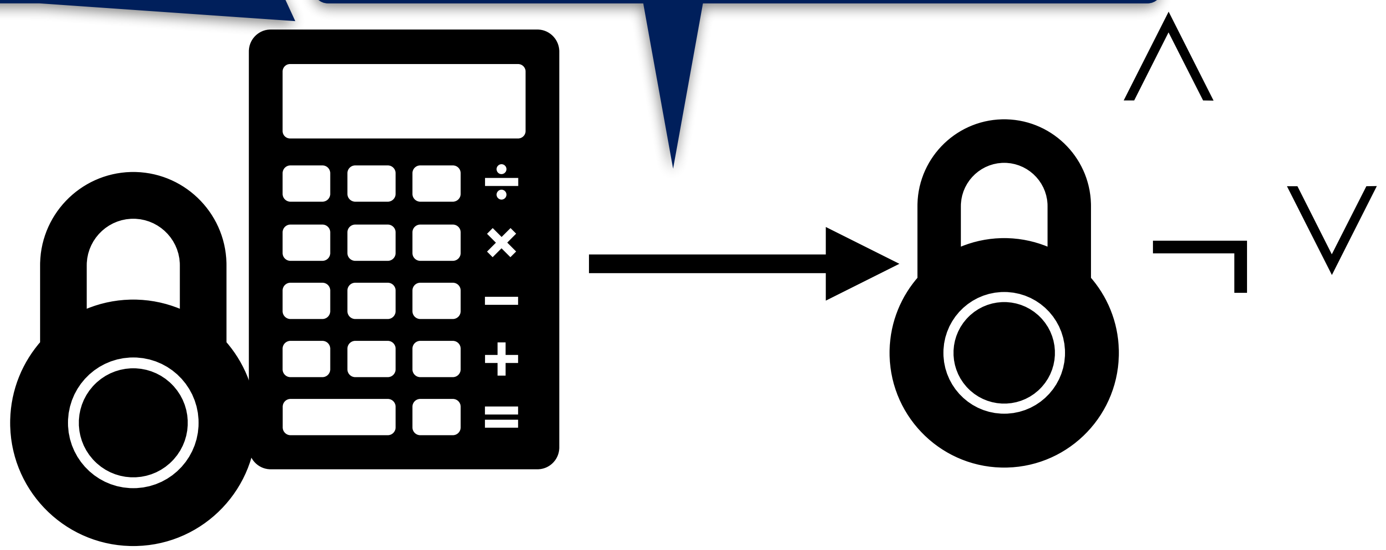


Oblivious Online Monitoring of STL with arithmetic predicates [Contribution]

1. Compute $\mu - c$ with CKKS

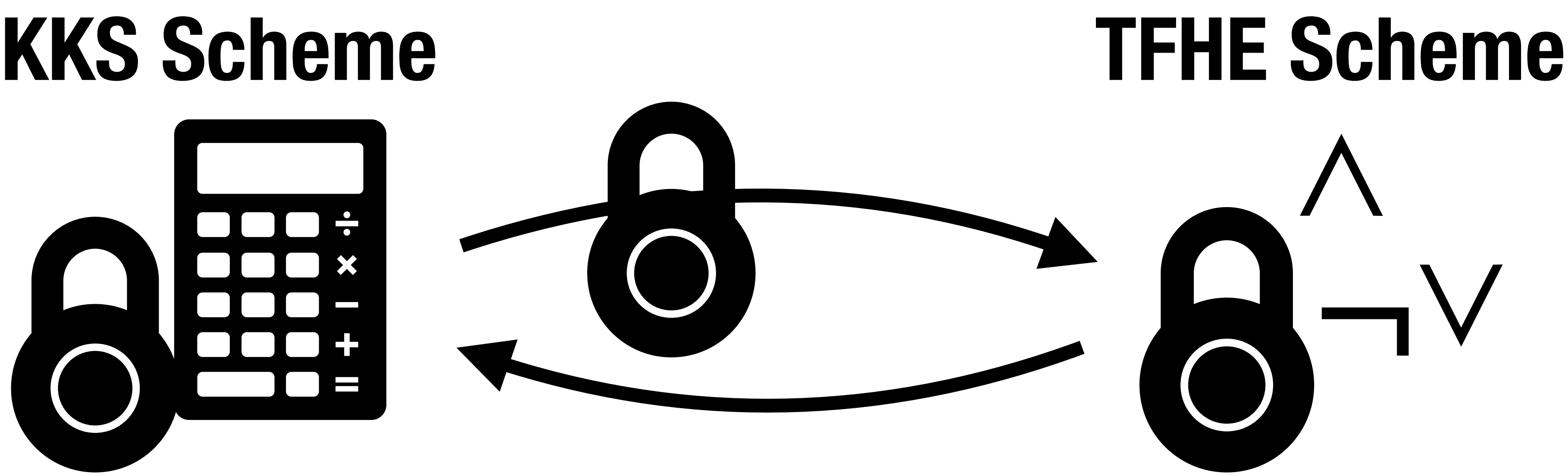
2. Obtain the truth value of $\mu > c$ as TFHE (sign of $\mu - c$)

3. Execute DFA with TFHE



Challenge: Scheme switch. is slow

Scheme switching: (Essentially) homomorphic re-encryption



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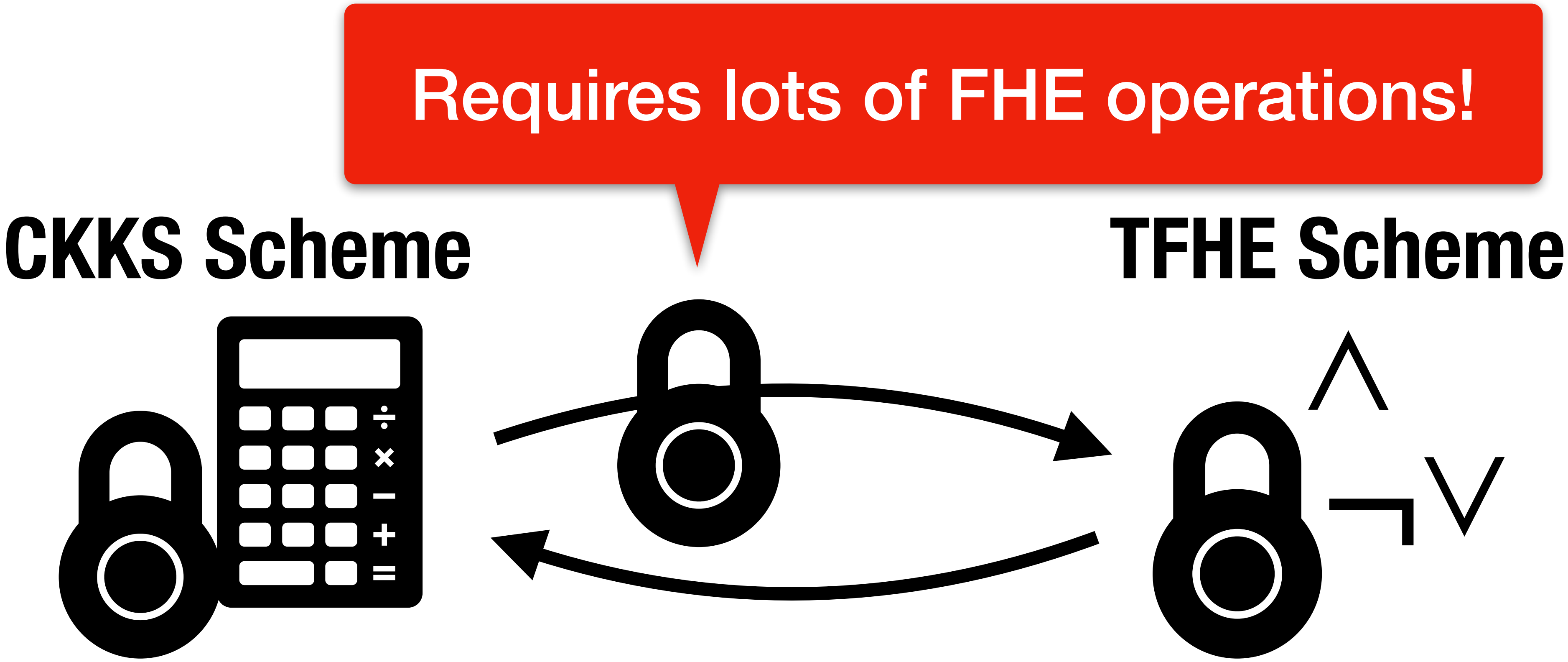

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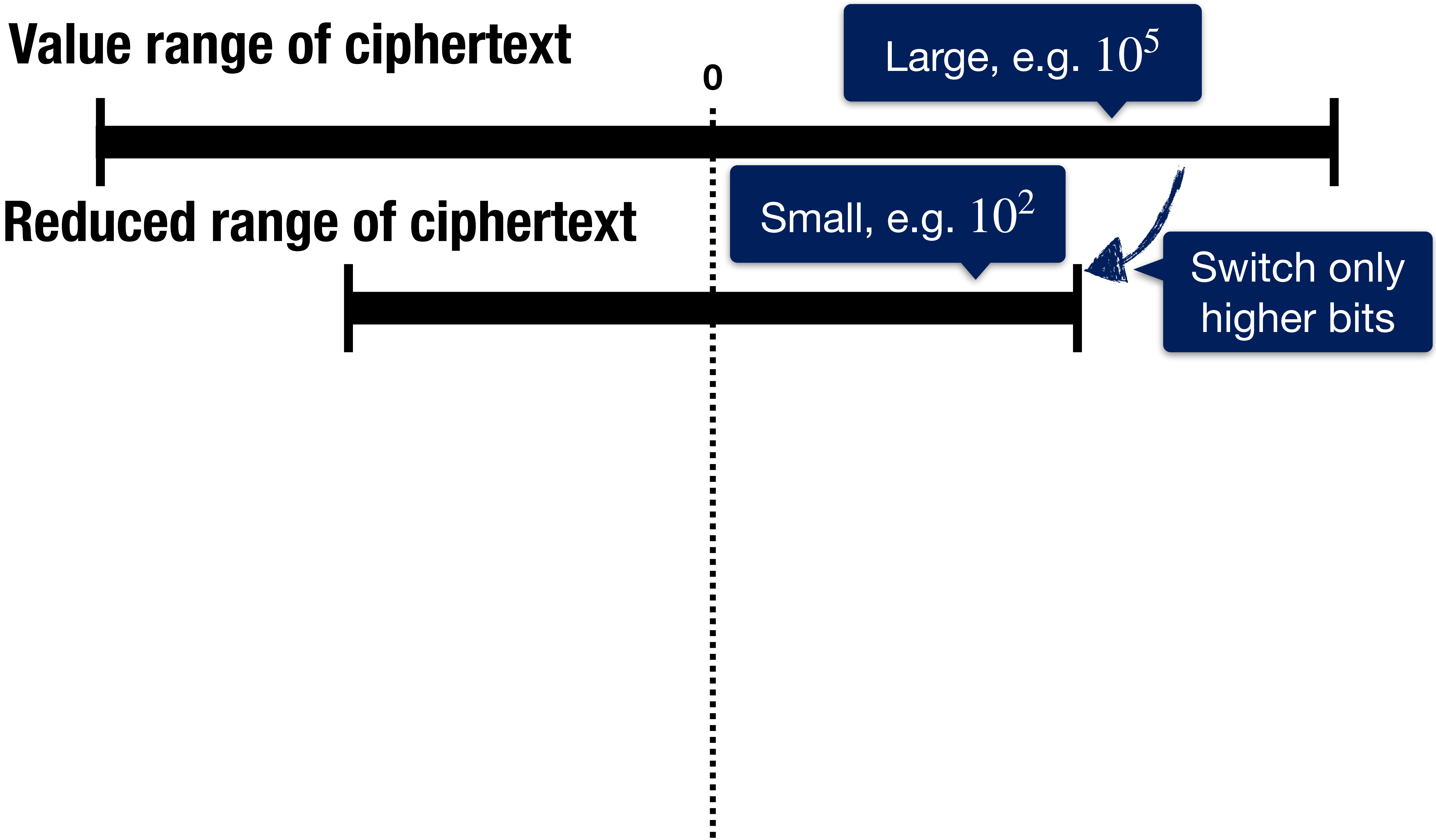
Optimization: Partial scheme switch



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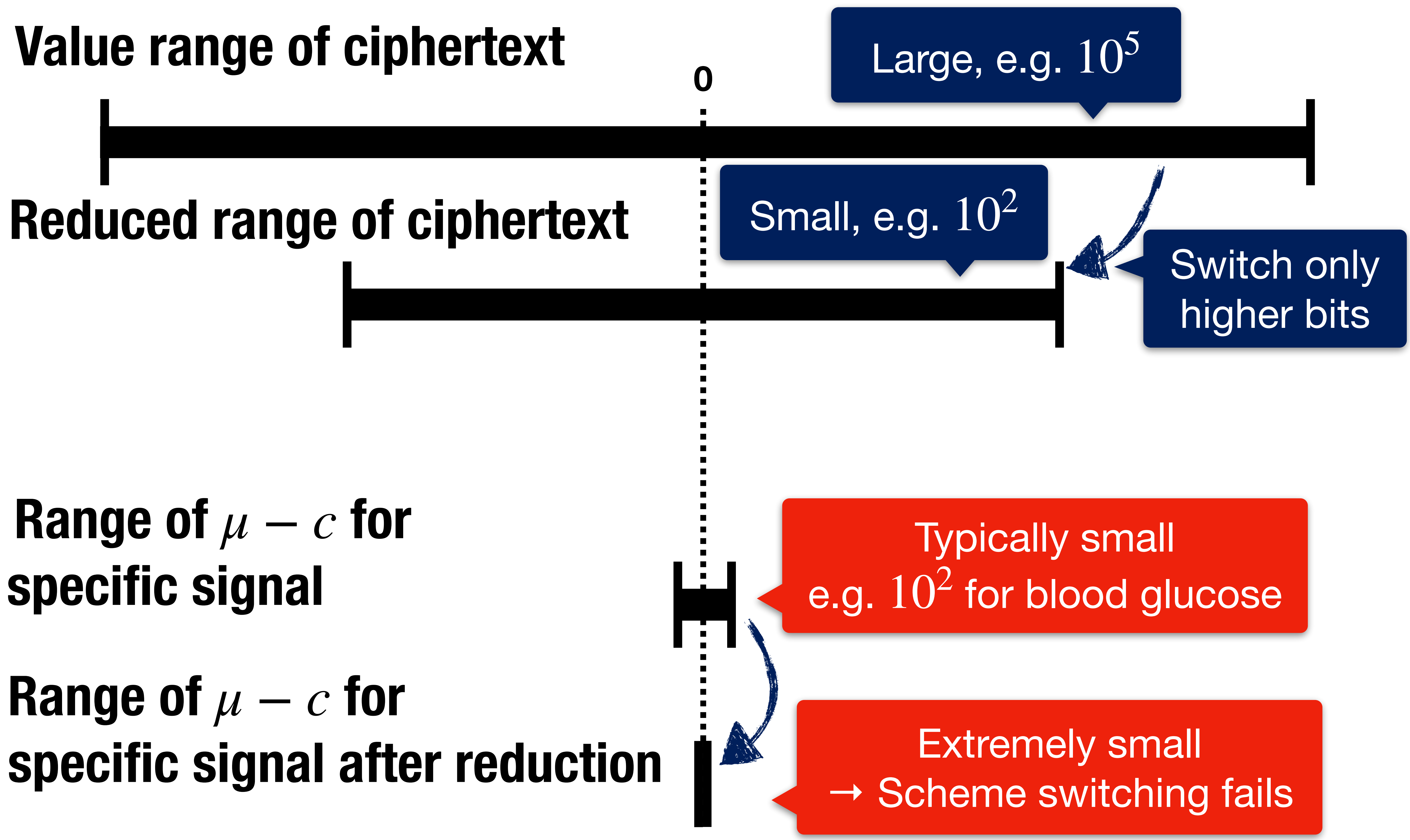
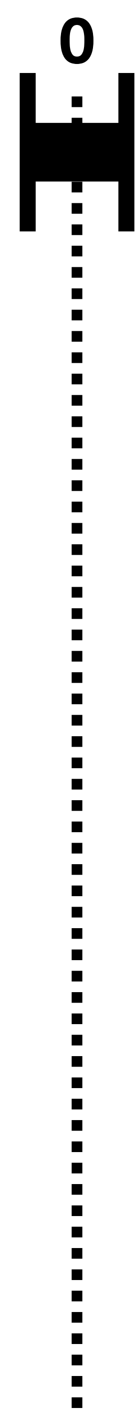


Optimization: Partial scheme switch

Range of $\mu - c$ for specific signal



Typically small
e.g. 10^2 for blood glucose

Optimization: Partial scheme switch

Range of $\mu - c$ for specific signal

0

Typically small
e.g. 10^2 for blood glucose

Amplify $\mu - c$
using signal range

Typically available from
domain knowledge

Range of $\mu - c$ for specific signal after scaling

Range of $\mu - c$ for specific signal after scaling & reduction

Large enough for
scheme switching

Complexity Analysis

Linear time and constant space wrt. log length
→ Scalable for online monitoring

	TFHE operations	CKKS Operations	Scheme Switching
Block	$O(n2^{\|\varphi\|})$	$O(n\|\varphi\|)$	$O(n\|\varphi\|)$
Reverse	$O(n2^{2^{\|\varphi\|}})$	$O(n\|\varphi\|)$	$O(n\|\varphi\|)$

New parts are linear to $|\varphi|$

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Setting of Experiments

- Implemented in C++ based on (Banno, CAV'22)
  - Microsoft SEAL for CKKS, TFHEpp for TFHE
- Benchmarks: Blood Glucose (Banno, CAV'22) and RSS
- AWS EC2 c6i.2xlarge (8 Core 16 GiB RAM) with Ubuntu 22.04

Overall Results

	Block		Reverse	
	Exec. Time	DFA Size	Exec. Time	DFA Size
$BGLvl_1$	323 ms/value	703	659 ms/value	172,402
$BGLvl_2$	318 ms/value	703	660 ms/value	172,402
$BGLvl_4$	254 ms/value	703	**OOM**	**OOM**
$BGLvl_5$	458 ms/value	72,603	**OOM**	**OOM**
$BGLvl_6$	519 ms/value	72,603	**OOM**	**OOM**
$BGLvl_7$	393 ms/value	3	290 ms/value	3
$BGLvl_8$	384 ms/value	5	300 ms/value	5
$BGLvl_{10}$	363 ms/value	27	248 ms/value	27
$BGLvl_{11}$	346 ms/value	27	262 ms/value	27
RSS	569 ms/value	179	511 ms/value	218

Overall Results

Fast enough for Blood Glucose (typical sampling rate > 1 min.)

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Maybe acceptable for less safety critical usage, e.g. reducing traffic jam

Optimized Scheme Switching

Our optimization reduces exec. time about 30%

	Optimized		Naive	
	Block	Reverse	Block	Reverse
$BGLvl_1$	323 ms/value	659 ms/value	500 ms/value	850 ms/value
$BGLvl_2$	318 ms/value	660 ms/value	517 ms/value	858 ms/value
$BGLvl_4$	254 ms/value	OOM	455 ms/value	OOM
$BGLvl_5$	458 ms/value	OOM	636 ms/value	OOM
$BGLvl_6$	519 ms/value	OOM	650 ms/value	OOM
$BGLvl_7$	393 ms/value	290 ms/value	585 ms/value	501 ms/value
$BGLvl_8$	384 ms/value	300 ms/value	600 ms/value	505 ms/value
$BGLvl_{10}$	363 ms/value	248 ms/value	539 ms/value	423 ms/value
$BGLvl_{11}$	346 ms/value	262 ms/value	536 ms/value	419 ms/value
RSS	569 ms/value	511 ms/value	943 ms/value	789 ms/value

Comparison with (Banno et al., CAV'22)

Scheme switching is very slow
→ (Banno et al., CAV'22) is faster if it works

	Ours		(Banno et al., CAV'22)	
	Block	Reverse	Block	Reverse
$BGLvl_1$	323 ms/value	659 ms/value	102 ms/value	OOM
$BGLvl_2$	318 ms/value	660 ms/value	102 ms/value	OOM
$BGLvl_4$	254 ms/value	OOM	28.4 ms/value	OOM
$BGLvl_5$	458 ms/value	OOM	OOM	OOM
$BGLvl_6$	519 ms/value	OOM	OOM	OOM
$BGLvl_7$	393 ms/value	290 ms/value	95.0 ms/value	0.876 ms/value
$BGLvl_{10}$	363 ms/value	248 ms/value	111 ms/value	5.54 ms/value
$BGLvl_{11}$	346 ms/value	262 ms/value	114 ms/value	8.84 ms/value

Comparison with (Banno et al., CAV'22)

Scheme switching is very slow
→ (Banno et al., CAV'22) is faster if it works

	Ours		(Banno et al., CAV'22)	
	Block	Reverse	Block	Reverse
$BGLvl_1$	323 ms/value	659 ms/value	102 ms/value	OOM
$BGLvl_2$	318 ms/value	660 ms/value	102 ms/value	OOM
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Comparison with (Banno et al., CAV'22)

Monitored value: numeric message instead of bit seq.
→ Smaller DFA

	Ours		(Banno et al., CAV'22)	
	Block DFA	Reverse DFA	Block DFA	Reverse DFA
$BGLvl_1$	703	172,402	10,524	**OOM**
$BGLvl_2$	703	172,402	11,126	**OOM**
$BGLvl_4$	703	**OOM**	7,026	**OOM**
$BGLvl_5$	72,603	**OOM**	**OOM**	**OOM**
$BGLvl_6$	72,603	**OOM**	**OOM**	**OOM**
$BGLvl_7$	3	3	21	20
$BGLvl_{10}$	27	27	237	237
$BGLvl_{11}$	27	27	390	390

Conclusions & Future Directions

- Online oblivious STL monitoring protocol
  - Combining CKKS and TFHE schemes
- Optimization of scheme switching for STL monitoring
- Implementation + experiments
  → Fast enough for blood glucose monitoring
  Works for RSS monitoring, too

Future directions

- Monitoring of *analog-digital mixed* signals
- Practical case study

Appendix

Ciphertexts for FHE: (R)LWE

Secret key | Raw plaintext

Ciphertext: (a, b) s.t. $b = s \cdot a + p + e$

Random "mask" | Random "noise"

- Random "noise" makes attack hard
- Each FHE operation amplifies noise
  e.g. × 2 by addition (on average)
- If $p + e \approx p$, we can decrypt correctly (at least approx.)
  → We need to keep noise small
  e.g., by Bootstrapping or using large ciphertext

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Formulas in Experiments

BGLvl_1	$\square_{[100,700]}(glucose \geq 70)$
BGLvl_2	$\square_{[100,700]}(glucose < 350)$
BGLvl_4	$\square_{[600,700]}(glucose < 200)$
BGLvl_5	$\neg\Diamond_{[200,600]}\square_{[0,180]}(glucose \geq 240)$
BGLvl_6	$\neg\Diamond_{[200,600]}\square_{[0,180]}(glucose < 70)$

BGLvl_7	$\square(glucose \geq 70 \wedge glucose < 180)$
BGLvl_8	$\square(\Delta glucose \geq -5 \wedge \Delta glucose < 3)$
BGLvl_{10}	$\square(glucose < 60 \Rightarrow \Diamond_{[0,25]} glucose \geq 60)$
BGLvl_{11}	$\square(glucose > 200 \Rightarrow \Diamond_{[0,25]} glucose < 200)$
RSS	$\square(S \wedge \mathcal{X}\neg S \Rightarrow \mathcal{X}(\varphi_{preBr} \wedge \varphi_{Br}))$

Client's Cost

Execution Time

	Enc. w/ public key [ms/value]	Enc. w/ private key [ms/value]	Decryption [ms/ciphertext]
NanoPi R6S (*w/* AES accelerator)	6.82	2.21	1.17×10^{-3}
Raspberry Pi 4 (*w/o* AES accelerator)	12.7	4.44	1.72×10^{-3}

Memory Usage

	Enc. w/ public key	Enc. w/ private key	Dec.
NanoPi R6S (*w/* AES accelerator)	360 656 kB	298 951.2 kB	6876.8 kB
Raspberry Pi 4 (*w/o* AES accelerator)	360 704 kB	299 089.6 kB	7168 kB

Detailed Experiment Results

	DFA eval. (sec.)				CKKS to TFHE (sec.)				CKKS eval. (sec.)				Runtime (sec.)			
	ArithHomFA$_{Opt}$		ArithHomFA$_{Naive}$		ArithHomFA$_{Opt}$		ArithHomFA$_{Naive}$		ArithHomFA$_{Opt}$		ArithHomFA$_{Naive}$		ArithHomFA$_{Opt}$		ArithHomFA$_{Naive}$	
	Block	Reverse	Block	Reverse	Block	Reverse	Block	Reverse	Block	Reverse	Block	Reverse	Block	Reverse	Block	Reverse
BGLvl$_1$	6.09e+01	2.99e+02	6.10e+01	3.02e+02	1.72e+02	1.76e+02	2.99e+02	3.12e+02	5.77e-02	5.55e-02	5.51e-02	5.32e-02	2.33e+02	4.75e+02	3.60e+02	6.13e+02
BGLvl$_2$	6.02e+01	3.01e+02	6.48e+01	3.00e+02	1.69e+02	1.76e+02	3.08e+02	3.18e+02	7.29e-02	7.01e-02	7.30e-02	6.99e-02	2.29e+02	4.76e+02	3.73e+02	6.18e+02
BGLvl$_4$	1.23e+01	**OOM**	1.33e+01	**OOM**	1.70e+02	**OOM**	3.15e+02	**OOM**	8.59e-02	**OOM**	8.67e-02	**OOM**	1.83e+02	**OOM**	3.28e+02	**OOM**
BGLvl$_5$	1.57e+02	**OOM**	1.56e+02	**OOM**	1.73e+02	**OOM**	3.02e+02	**OOM**	5.67e-02	**OOM**	5.34e-02	**OOM**	3.30e+02	**OOM**	4.59e+02	**OOM**
BGLvl$_6$	1.87e+02	**OOM**	1.59e+02	**OOM**	1.87e+02	**OOM**	3.09e+02	**OOM**	8.22e-02	**OOM**	7.13e-02	**OOM**	3.74e+02	**OOM**	4.68e+02	**OOM**
BGLvl$_7$	9.79e+02	1.18e+01	9.56e+02	1.13e+01	2.98e+03	2.91e+03	4.94e+03	5.04e+03	1.65e+00	1.95e+00	1.62e+00	1.91e+00	3.96e+03	2.93e+03	5.90e+03	5.06e+03
BGLvl$_8$	9.63e+02	1.24e+01	9.67e+02	1.28e+01	2.91e+03	3.01e+03	5.08e+03	5.07e+03	2.49e+00	3.17e+00	2.60e+00	3.10e+00	3.87e+03	3.03e+03	6.05e+03	5.09e+03
BGLvl$_{10}$	1.15e+03	1.23e+01	1.14e+03	1.16e+01	2.51e+03	2.48e+03	4.29e+03	4.25e+03	8.26e-01	8.87e-01	8.11e-01	8.39e-01	3.66e+03	2.50e+03	5.43e+03	4.26e+03
BGLvl$_{11}$	1.09e+03	1.25e+01	1.14e+03	1.13e+01	2.40e+03	2.63e+03	4.26e+03	4.22e+03	7.79e-01	9.81e-01	7.84e-01	8.49e-01	3.49e+03	2.65e+03	5.40e+03	4.23e+03
RSS	4.89e+00	5.11e-01	5.10e+00	4.94e-01	2.25e+01	2.41e+01	4.07e+01	3.77e+01	4.61e-01	4.77e-01	4.79e-01	4.60e-01	2.79e+01	2.50e+01	4.62e+01	3.87e+01

Detailed Experiment Results

Depends on DFA size

Slow

Fast

	DFA eval. (sec.)				CKKS to TFHE (sec.)				CKKS eval. (sec.)				Runtime (sec.)			
	ArithHomFA$_{\text{Opt}}$		ArithHomFA$_{\text{Naive}}$		ArithHomFA$_{\text{Opt}}$		ArithHomFA$_{\text{Naive}}$		ArithHomFA$_{\text{Opt}}$		ArithHomFA$_{\text{Naive}}$		ArithHomFA$_{\text{Opt}}$		ArithHomFA$_{\text{Naive}}$	
	Block	Reverse	Block	Reverse	Block	Reverse	Block	Reverse	Block	Reverse	Block	Reverse	Block	Reverse	Block	Reverse
BGLvl$_1$	6.09e+01	2.99e+02	6.10e+01	3.02e+02	1.72e+02	1.76e+02	2.99e+02	3.12e+02	5.77e-02	5.55e-02	5.51e-02	5.32e-02	2.33e+02	4.75e+02	3.60e+02	6.13e+02
BGLvl$_2$	6.02e+01	3.01e+02	6.48e+01	3.00e+02	1.69e+02	1.76e+02	3.08e+02	3.18e+02	7.29e-02	7.01e-02	7.30e-02	6.99e-02	2.29e+02	4.76e+02	3.73e+02	6.18e+02
BGLvl$_4$	1.23e+01	OOM	1.33e+01	OOM	1.70e+02	OOM	3.15e+02	OOM	8.59e-02	OOM	8.67e-02	OOM	1.83e+02	OOM	3.28e+02	OOM
BGLvl$_5$	1.57e+02	OOM	1.56e+02	OOM	1.73e+02	OOM	3.02e+02	OOM	5.67e-02	OOM	5.34e-02	OOM	3.30e+02	OOM	4.59e+02	OOM
BGLvl$_6$	1.87e+02	OOM	1.59e+02	OOM	1.87e+02	OOM	3.09e+02	OOM	8.22e-02	OOM	7.13e-02	OOM	3.74e+02	OOM	4.68e+02	OOM
BGLvl$_7$	9.79e+02	1.18e+01	9.56e+02	1.13e+01	2.98e+03	2.91e+03	4.94e+03	5.04e+03	1.65e+00	1.95e+00	1.62e+00	1.91e+00	3.96e+03	2.93e+03	5.90e+03	5.06e+03
BGLvl$_8$	9.63e+02	1.24e+01	9.67e+02	1.28e+01	2.91e+03	3.01e+03	5.08e+03	5.07e+03	2.49e+00	3.17e+00	2.60e+00	3.10e+00	3.87e+03	3.03e+03	6.05e+03	5.09e+03
BGLvl$_{10}$	1.15e+03	1.23e+01	1.14e+03	1.16e+01	2.51e+03	2.48e+03	4.29e+03	4.25e+03	8.26e-01	8.87e-01	8.11e-01	8.39e-01	3.66e+03	2.50e+03	5.43e+03	4.26e+03
BGLvl$_{11}$	1.09e+03	1.25e+01	1.14e+03	1.13e+01	2.40e+03	2.63e+03	4.26e+03	4.22e+03	7.79e-01	9.81e-01	7.84e-01	8.49e-01	3.49e+03	2.65e+03	5.40e+03	4.23e+03
RSS	4.89e+00	5.11e-01	5.10e+00	4.94e-01	2.25e+01	2.41e+01	4.07e+01	3.77e+01	4.61e-01	4.77e-01	4.79e-01	4.60e-01	2.79e+01	2.50e+01	4.62e+01	3.87e+01